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International Correspondence Schools, Scranton, Pa.

Mapping

Prepared Especially for Home Study

By

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2913 A-3
4 Assignments

Part 1

Edition 1

International Correspondence Schools, Scranton, Pennsylvania

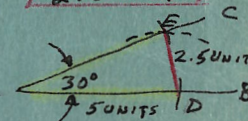
International Correspondence Schools, Canadian, Ltd., Montreal, Canada

$$2(AD) \frac{\sin \alpha}{2}$$

$$2(5 \text{ UNITS}) \frac{.500}{2} =$$

$$(10 \text{ UNITS})(.250) =$$

$$2.5 \text{ UNITS}$$



"The beginning is half the thing."
—Greek Proverb

* * *

"Well begun is half done" is another familiar saying. In your decision to "do something" to add to your fund of knowledge through some systematic spare-time study, you are "well begun." And if you will persist in this determination, diligently studying one assignment after another, one day, not too far in the future, you'll "BE DONE."

Harold G. Davis

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What This Text Covers . . .

1. PRELIMINARY EXPLANATIONS Pages 1 to 28

In this introductory section you learn about scales of maps, methods of designating directions of lines, the use of instruments for plotting parallel lines, methods of plotting angles, the factors in planning maps, and methods of forming the roman style of letters.

2. PLOTTING TRAVERSES Pages 29 to 59

This section deals with the methods of locating points on traverses and includes complete instructions for preparing two drawing plates. One plate covers several methods of plotting open traverses and the other plate shows the application of two methods of plotting closed traverses.

3. ROUTES INVOLVING CIRCULAR CURVES Pages 60 to 79

The first part of this section explains the basic principles relating to circular curves. Then instructions are given for preparing a drawing plate involving the plotting of two routes for highway or railroad location by different methods.

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NOTICE

For an explanation of the letters placed on your drawings, refer to *Key to Criticisms* at the end of this paper.

MAPPING

PART I

PRELIMINARY EXPLANATIONS

INTRODUCTION

1. **Kinds of Maps.**—A map is a representation, by means of lines and symbols, of a portion of the surface of the earth. The positions of the points and the directions and lengths of the lines that are shown on a map are established by making a survey of the area that is to be mapped. Usually, the map is drawn on a flat surface. Also, the earth's surface is generally assumed to be projected onto a horizontal plane. Therefore, the distance between any two points on the map is understood to represent the horizontal distance between the corresponding points on the earth's surface. There are also maps on which allowance is made for the fact that the earth is essentially spherical in shape, but the construction of such maps is not included in these texts on *Mapping*.

Maps may be prepared for many different purposes, and the amount of information that should be included on a particular map is determined largely by its purpose. A very simple type of map is a property map that is intended to show merely the size and shape of a single farm or of the several tracts of a subdivision. Such a map will contain little more than the lines that represent the boundaries, or limits, of the property involved. On another type of map are represented the property lines and also various material objects that are located in the area covered by the map. Real-estate display maps are often of this class. On a third type of map are indicated not only the property lines and the material objects, but also the *relief*, or the variations in elevation of the ground surface. In this group are geographic maps, general topographic maps, and maps for engineering projects of many kinds.

2. Required Drawing Plates.—The primary objectives of this instruction in mapping are to present the principles and explain the methods that are applied in the preparation of maps of various kinds, and also to show how the information contained in the field notes of a survey is utilized in preparing a map. For the purpose of illustrating the procedure that is followed in practical work, seven drawing plates are included in the texts on *Mapping*, Parts 1 and 2—three in Part 1 and four in Part 2. Each of these plates involves one or more types of problems in plotting that are commonly encountered in engineering practice. In each case, the student is required to prepare a map in accordance with data and instructions given in the text.

3. Tracings and Other Reproductions.—The preparation of a map often involves a considerable amount of work, and the original map may therefore be very valuable. Also, the original drawing must usually be preserved for record and reference, but the information shown on it may be of interest to many persons. If every one of the interested persons were permitted to use the original drawing, it would soon become damaged and badly worn—especially if ordinary drawing paper is used—and it might even be lost.

These difficulties can be readily overcome by making reproductions of the original drawing. Usually, a copy of the map is made on *tracing cloth*, which is linen that has been specially treated with a waxy sizing in order to make it transparent. The tracing cloth is placed over the original drawing, and the lines, which may be plainly seen, are traced in ink on the surface of the cloth. Such copies, which are called tracings, are more durable than paper drawings, but their chief advantage is that as many reproductions of a drawing as may be desired can be obtained from a single tracing. The reproductions commonly used are made by a process known as *blueprinting*. A blueprint is a reproduction of the drawing in white lines on a blue background; it is produced by placing the tracing over a piece of blueprint paper, exposing both to bright light for a short time, and then washing the blueprint paper in water.

It is often preferable in mapping work to make blue-line or brown-line prints, in which the lines and lettering are in blue or brown and the background is white. Such prints are more expensive than blueprints, but they can be read more easily, and notes and alterations that are made on the print can be readily seen. Occasionally, it is desirable to use reproductions known as *lithographs*, which are black-line prints on white paper; or *photostats*, which are photographic copies of the original drawing.

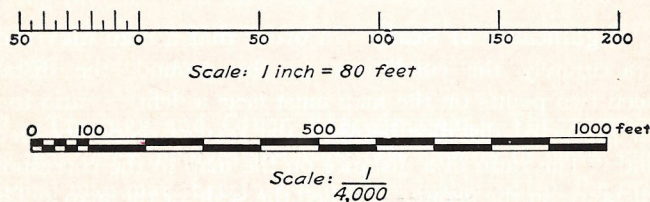
SCALE OF MAP

4. Significance of Scale.—In order that a map may represent accurately the conditions on the ground, the distance between two points on the map must bear a definite ratio to the actual projected distance between the respective points on the ground. The ratio of a distance on the map to the corresponding distance on the ground is called the scale of the map. Thus, if 1 inch on the map represents 100 feet on the ground, the scale of the map is $\frac{1}{12} \div 100$, or $\frac{1}{1200}$. In the case of a map on which all the projected distances are horizontal, the scale is ordinarily made the same in all directions in a horizontal plane. Hence, if the scale of the map is known, it is a simple matter to determine either what distance on the map represents a certain measured distance on the ground, or what distance on the ground corresponds to a distance measured on the map.

5. Selection of Scale.—The proper scale of a map depends on the purpose of the map and on the size of the area to be mapped. The scale should always be large enough to permit all necessary details to be shown clearly without making the map unwieldy. For engineering maps of building sites, a scale of 1 inch = 20 feet or even 1 inch = 10 feet may be necessary to show all the details with sufficient accuracy. On the other hand, a scale of about 1 inch = 15 miles may be used for a map of a very large area.

6. Methods of Indicating Scale.—It is always essential to specify on a map the scale to which the map is drawn. The

scale may be indicated graphically, as in Fig. 1, or may be specified by means of a suitable note, such as Scale: $1'' = 100'$, or Scale: $\frac{1}{1200}$. In general, a graphic scale consists of a distance that is subdivided into a number of parts each of which represents a certain stated distance on the ground. Thus, on the first of the two scales here illustrated, each large division represents a distance of 50 feet on the ground, and each small subdivision of the 50-foot division at the left-hand end represents 5 feet. On the other scale, each large division represents 100 feet and each small subdivision represents 20 feet.



GRAPHIC SCALE FIG. 1

A noted scale has the advantage of simplicity, as special measuring rules or scales are available by means of which distances can be readily measured to a given scale. Therefore, where a graphic scale is used, it is customary to include also on the original drawing a noted scale, as in Fig. 1. However, if the paper on which the map is drawn should shrink or stretch, the noted scale no longer applies. Also, it is often desirable to make a photostatic copy of the map to a smaller size. In such a case, the ratio in the noted scale would appear on the photostat exactly as on the original map, whereas the actual scale of the reproduction would be materially different from that of the original map. On the other hand, the length of each division of a graphic scale changes proportionately with any variation in the size of the original map or with a reduction by photography or any other similar process. Before a map that contains both a graphic and a noted scale is photographed to a different scale, the noted scale is generally blocked out or removed.

7. Engineers' Scale.—The type of measuring scale that is commonly used in mapping is the engineers' scale. Usually, it

is triangular in shape, as shown in Fig. 2, is 12 inches in length, and is made of boxwood. Triangular scales that are 6, 18, or 24 inches long are also obtainable. There are also flat scales, which have lengths of 6 or 12 inches. The scale shown in Fig. 2 has a different system of graduation on each side of each edge; therefore, it is actually a combination of six scales. Each scale is so divided that the number of divisions in 1 inch is a multiple of ten, this number being indicated by large figures in the center of the scale. Thus, the numbers 10 and 50 on the scales that are visible indicate that these scales are divided, respectively, to tenths and fiftieths of an inch. Every tenth graduation mark is long, and the number of long graduations, or main divisions, from the zero of the scale is shown by the figures. Hence, the figure 32 on the 50-scale indicates a distance of 32 main divisions or $32 \times 10 = 320$ subdivisions from zero; the actual distance in inches is unimportant.

The scales most commonly provided on the triangular form of engineers' scale are those with 10, 20, 30, 40, 50, and 60 divisions to the inch. The 10-scale is convenient for plotting to a scale such as 1 inch = 10 feet, 1 inch = 100 feet, or 1 inch = 1,000 feet, because each subdivision then represents 1 foot, 10 feet, or 100 feet, as the case may be. Similarly, the 20-scale is suited to plotting to a scale such as 1 inch = 20 feet or 1 inch = 200 feet; and the other edges of the ruler are adapted to scales corresponding to the respective numbers of divisions per inch or to simple multiples of those numbers. Fractional parts of the divisions on any scale can be estimated by eye.

8. Use of Engineers' Scale.—The procedure for laying off a certain distance with the engineers' scale may be outlined as

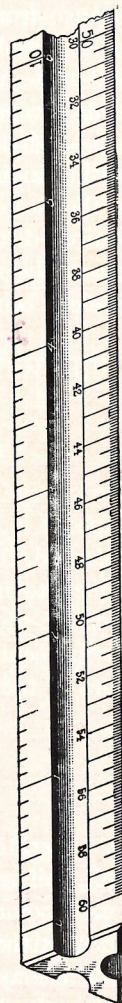


FIG. 2

follows: With the zero mark of the proper scale at the beginning of the measurement and the edge of the scale in the correct direction, a new point is marked on the map at the required distance from the zero mark. For example, a distance of 283 feet to a scale of 1 inch = 200 feet is laid off as shown in Fig. 3. The zero mark of the 20-scale is first placed opposite the point *A* representing the beginning of the measurement. Since each subdivision on the scale represents 10 feet and each main division indicates 100 feet, the graduation opposite the figure 2 on the scale indicates 200 feet. Hence, the point *B* at the required distance of 283 feet from *A* is located 8.3 subdivisions beyond the second long graduation, the decimal part of a subdivision being estimated by eye. Where great accuracy is desired, a needle may be used for marking the positions of points on the map.

When reading the scale, the observer should be careful to have his eye directly over the point from or to which the measurement is being made. Before a distance is read or a point is marked, the position of the zero mark of the scale should be checked to make sure that it has not been moved. Also, the pencil or the needle used to mark the point at the end of a measurement should be held in a vertical plane that is at right angles to the edge of the scale.

FIG. 3

DIRECTIONS OF LINES

METHODS OF DESIGNATING DIRECTIONS

9. **Meridians.**—The directions of the lines of a survey are usually referred to some fixed line, which is called a meridian. At each point on the earth's surface, there are two definite lines, known as the true meridian and the magnetic meridian, either of which may be used for reference. The true meridian at any point is the line that passes through the point and is directed toward the north and south geographic poles of the earth. The magnetic meridian is the line passing through the point and directed toward the north and south magnetic poles. In some

LONGITUDE
LATITUDE

localities the magnetic and true meridians may coincide, but usually the two meridians have somewhat different directions.

Actually, true and magnetic meridians are circles on the earth's surface and all meridians of each type meet at the respective poles. On maps of relatively small areas, meridians are treated as parallel straight lines that lie in a horizontal plane, and the meridian at the starting point of the survey is assumed to govern the directions of all lines of the survey. However, on maps of large areas, it may be necessary to indicate the direction of the meridian at regular intervals.

10. **Azimuths.**—The azimuth of a line is the angle, measured in a clockwise direction, from the meridian to the line.

The angle is a *true azimuth* if measured from a true meridian, and is a *magnetic azimuth* if measured from a magnetic meridian. Azimuths range from 0° to 360°. They may be measured from either the north end or the south end of the meridian. In Fig. 4, *NS* represents a meridian with *N* toward the north. Then the azimuths, measured from the north, of *OA*, *OB*, and *OC* are, respectively, 115°, 246°, and 300°.

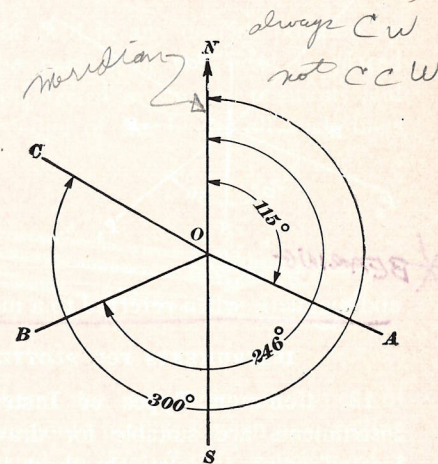


FIG. 4

11. **Bearings.**—Another method of designating the angle between a meridian and a line is by the bearing of the line, which is the smallest angle between the meridian and the line and may be measured either clockwise or counter-clockwise from the meridian. In determining the bearings, the plane around any given point is divided into four quadrants by two lines, of which one is a meridian, or north-and-south line, through the point and the other is an east-and-west line. Bear-

ings are reckoned from 0° to 90° in each quadrant, the zero bearings being in the meridian and the 90° bearings in the east-and-west line. Thus, through any given point it is possible to draw four lines, one in each quadrant, all of which make the same angle with the meridian. In order to distinguish between these lines, the letters *N*, *E*, *S*, and *W*, representing north, east, south, and west, respectively, are used to indicate the quadrant. If a line is in the northeast quadrant, its bearing is written with the letter *N* preceding the value of the angle, and the letter *E* following the angle. For a line in the southeast quadrant, the letter *S* precedes the angle and *E* follows. Lines in the other

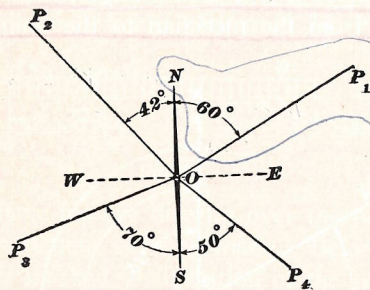


FIG. 5

quadrants are indicated by the corresponding letters in a similar manner. For example, in Fig. 5, the bearing of the line OP_1 is *N* 60° *E*; that of OP_2 is *N* 42° *W*; that of OP_3 is *S* 70° *W*; and that of OP_4 is *S* 50° *E*.

A bearing is true when referred to a true meridian, and magnetic when referred to a magnetic meridian.

INSTRUMENTS FOR PLOTTING PARALLEL LINES

12. **Common Types of Instruments.**—Various types of instruments are suitable for drawing straight lines between located points. Relatively short lines are conveniently drawn with a triangle. For longer lines, it is customary to use a long straightedge of steel or of wood with bakelite edges. Also, the **T** square and the parallel ruler are employed to a considerable extent for drawing parallel lines.

13. **T Square.**—The common style of **T** square has a solid head and a blade that is fixed at right angles to the head. Therefore, only parallel lines at right angles to an edge of the drawing board can be drawn with such an instrument. For drawing lines that are parallel to each other but are not perpendicular

to the edge of the board, the type of **T** square shown in Fig. 6 is convenient. Its head is divided into two parts, *a* and *b*. The part *a* is rigidly fixed at right angles to the blade *c* and is connected to the part *b* by means of a bolt and a thumbscrew. When the thumbscrew is loosened, the part *a* of the head can be rotated to the right or to the left with respect to the part *b*;

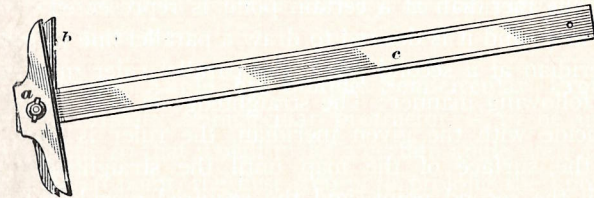


FIG. 6

and the two parts can then be held in any desired position by tightening the screw. If the part *b* of the head is placed against an edge of the drawing board in various positions and lines are drawn along either edge of the blade, all such lines will be parallel to each other.

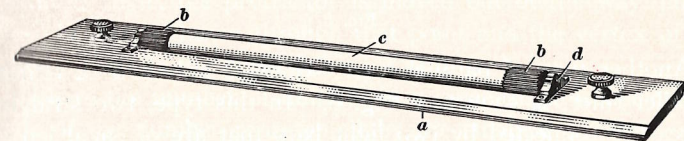


FIG. 7

14. **Parallel Rulers.**—There are two general types of parallel rulers. The type commonly used is known as a rolling parallel ruler and is illustrated in Fig. 7. It consists of a straightedge *a* which is mounted on two milled rollers *b*, that have equal diameters and are attached to a common shaft. Above the shaft is a hand hold or grip *c* for moving the ruler. The height of the supports *d* at the ends of the shaft is such that the rollers project about $\frac{1}{8}$ inch beyond the lower surface of the straightedge. When the rollers are in contact with a flat surface, the straightedge is therefore slightly above the surface and the instrument can be readily rolled over the surface. The rolling parallel rulers that are generally employed are made of metal and are fairly heavy.

If the ruler is carefully moved over a surface without lifting either roller, two or more lines drawn along the straightedge in different positions will be parallel to each other. Therefore, this instrument can be conveniently used for drawing parallel lines or for the purpose of transferring the direction of a line from one part of a map to another. For example, if the direction of the meridian at a certain point is represented by a line on the map, and it is desired to draw a parallel line to represent the meridian at a second point, the parallel ruler may be used in the following manner: The straightedge is first placed so as to coincide with the given meridian, the ruler is then rolled along the surface of the map until the straightedge passes through the second point, and the required meridian is established by drawing a line along the straightedge.

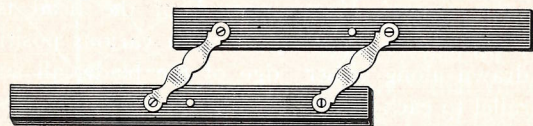


FIG. 8

Another type of parallel ruler, which is known as a folding parallel ruler, is shown in Fig. 8. In this type, two straightedges are connected by two light bars that are of equal length and are pivoted at the ends. Thus, the two straightedges are parallel to each other in all positions, but the distance between them can be altered. Such an instrument is suitable only for drawing parallel lines fairly close together.

PLOTTING ANGLES

15. Methods of Laying Off Angles.—An angle may be laid off on a map in any one of several ways. The simplest and most rapid method is by means of a protractor. But, where greater accuracy is desired, use is made of either the *tangent method* or the *chord method*. In the tangent method, the angle is plotted by constructing a right triangle in which the ratio of the lengths of the two perpendicular sides is equal to the natural tangent of the angle; in other words, the angle is measured by

its natural tangent. In the chord method, the angle is plotted by constructing an isosceles triangle in which the two equal sides are assumed radii of a circular arc and the third side is the chord of the arc for the given angle and the assumed radii; the desired angle is thus between the two equal sides of the triangle. The tangent and chord methods are equally convenient, and the choice between them depends entirely on the personal preference of the draftsman.

16. Plotting Angle by Simple Semicircular Protractor.

When an ordinary semicircular protractor is to be used for plotting a line so that it will make a certain angle with a known line at a given point, the first step is to prolong the known line on each side of that point, which will be the vertex of the required angle, for a distance that is slightly greater than the radius of the protractor. The next step is to place the protractor with its center exactly at the vertex of the angle and its diameter along the known line. Then, a point is marked opposite the graduation on the protractor corresponding to the required angle, the protractor is moved out of the way, and a straight line is drawn through that point and the vertex of the angle. This is the desired line.

Where the number of minutes in a given angle is not a multiple of the number represented by the smallest subdivision of the protractor, the required angle is not indicated by a graduation mark. The distance from a graduation mark to the point on the graduated arc corresponding to the given angle should then be estimated by eye and laid off as accurately as possible.

17. Protractor With Rotating Arm.—In Fig. 9 is shown a type of protractor that is better adapted for mapping than the simple semicircular protractor. It consists of a metal protractor with a rotating arm, or blade, *a*, which extends from the center of the graduated arc. In order that the protractor may be centered accurately at the desired point on the paper, the center is marked by cross lines on a transparent celluloid disk in the joint *b* by which the blade is attached. The arm *a* is provided with a vernier *c*, by means of which settings can be made to the nearest minute. The half-degree graduations are

omitted in this illustration in order to make the divisions clearer.

If a protractor with an arm is used for plotting, the procedure is somewhat similar to that described for a simple protractor. The center of the protractor is set over the vertex of the angle, the zero mark of the protractor is made to coincide with the known line or its prolongation, and the arm is set to the required angle. Then a line drawn along the edge of the arm has the desired direction. The arm can be set to read the angle either before or after the protractor is placed in the proper position on the map.

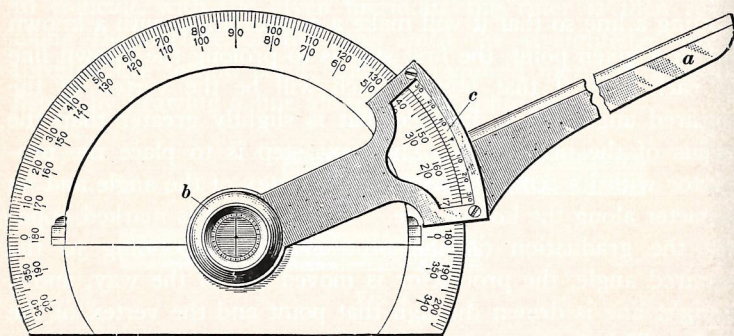


FIG. 9

18. Paper Protractor.—When a simple semicircular protractor or an instrument like that illustrated in Fig. 9 is used, the protractor must be centered over every point at which an angle has to be laid off. Thus, much time is required and there is likely to be some error in the plotting because of the difficulty of placing the protractor in the proper position at each point. The form of protractor known as a paper protractor is advantageously employed in plotting surveys in which a large number of angles are involved. Such a protractor is a graduated circle printed from an accurately engraved plate on a sheet of drawing paper, tracing paper, or bristol board. It can be tacked onto the drawing board and may be kept in one position for plotting many lines whose directions are referred to the meridian by bearings or azimuths, even though all the lines do not pass through the same point.

The two standard diameters of the graduated circles of paper protractors are 8 inches and 14 inches. The smallest divisions represent half-degrees on the 8-inch protractor and quarter-degrees on the 14-inch protractor. Paper protractors may be obtained with or without the numbers of the graduations printed on the sheets. If the values of the angles are printed, they run from 0° to 360° clockwise, and the protractor is especially convenient for plotting by azimuths. In the case of unnumbered sheets, the values of the angles may be inserted by the draftsman in the manner that is most convenient for his purpose. Thus, for azimuths the graduations are numbered from 0° to 360° , clockwise, and for bearings they are numbered in quadrants from 0° to 90° in each direction from the assumed north-and-south line.

19. Use of Paper Protractor.—When the directions of the lines on a map are to be determined by the use of a paper protractor, the first step is to draw, in some convenient position on the map, a long line to represent the direction of the meridian. At any place along this meridian line, the protractor is tacked onto the drawing board in such a position that the two zero marks, or the 0° and 180° marks, on the protractor will lie on the meridian line. A line having a specified azimuth or bearing and passing through the point that is marked by the center of the protractor can then be located by setting a straight-edge so as to pass through the center of the protractor and also through the proper graduation mark on the circle. If the straightedge passes through two graduation marks that are diametrically opposite, it will necessarily pass through the center of the protractor and a little greater accuracy will be obtained.

The center of the protractor need not be over the point through which a line having a certain desired direction is to be drawn. A direction may be readily transferred from the center of the protractor to another point on the map by using as the straightedge either a parallel ruler or a T square of the type illustrated in Fig. 6. If neither of these instruments is available, two large triangles can be employed in the same manner as for drawing parallel lines. An edge of the ruler, T square,

or triangle is first set so as to pass through the center of the protractor and the point on the graduated circle that corresponds to the desired azimuth or bearing. Then, the straightedge is moved parallel to itself to the point on the map through which the line is to be plotted.

20. In order to illustrate the method of plotting lines by the use of a paper protractor and a parallel ruler, the procedure will be described for plotting three lines that are represented in Fig. 10 by AB , BC , and CD . The bearings of these lines are

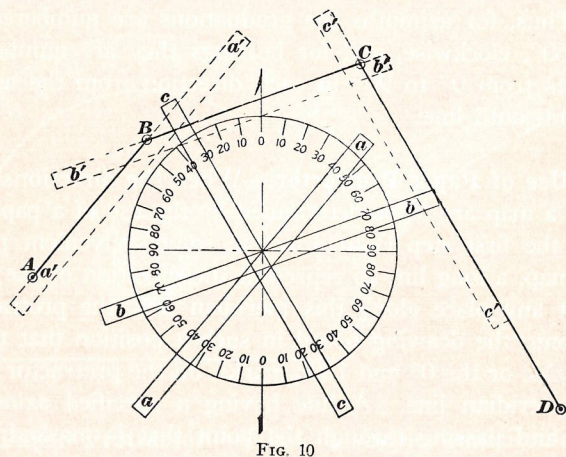


FIG. 10

as follows: AB , N 40° E; BC , N 70° E; and CD , S 30° E. The meridian is represented by a vertical line with a half arrow-head, north being toward the top of the page as indicated. The graduations on the protractor are numbered for reading bearings, and only the 10° graduations are shown.

The plotting is begun by locating the point A in a convenient position on the paper and marking this position by means of a needle hole with a small circle around it. Then the parallel ruler is set on the protractor in the position aa , that is, so that one straightedge reads the bearing of the line AB , or N 40° E. The next operation consists in rolling the ruler to the position $a'a'$, where the edge passes through the point A , and to draw a line in the direction of the point B . On this line, the length of

the line AB is laid off to the scale used for the map and the point B is marked with a needle hole and a circle. To locate point C , the parallel ruler is first placed across the protractor in the position bb , so that the straightedge reads the bearing of the line BC , or N 70° E. It is then rolled to the position $b'b'$, so that the edge passes through the point B , and a line is drawn toward C . The distance BC is laid off to scale on this line, and the point C is marked. The last point D is located in a similar manner. Thus, the ruler is first placed in the position cc , or so as to read a bearing of S 30° E; and is then rolled to the position $c'e'$, or so as to pass through C . Finally, the line is drawn toward D and the distance CD is laid off along it to scale.

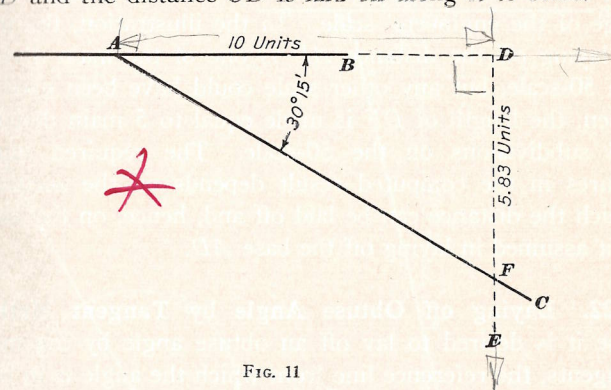


FIG. 11

21. **Laying off Acute Angle by Tangent Method.**—When the method of tangents is to be applied for locating a line that makes a given acute angle with a reference line at a certain point, the procedure is as indicated in Fig. 11. Here, AB is the reference line, and it is desired to plot a line AC that makes a certain clockwise angle with AB at the point A . The first step is to lay off, from the vertex at A , any convenient distance AD along the reference line or its prolongation. This distance is the base of a right triangle of which the required angle is a part. The next step is to erect a perpendicular DE to the reference line at point D . This perpendicular is extended in the direction in which the angle is to be turned. The third vertex of the right triangle that determines the direction of the required line is then located by laying off along the perpen-

dicular DE a distance DF equal to the product of the assumed base AD of the triangle and the natural tangent of the given angle. The straight line AC , which is drawn through the given point A and the point F , makes the required angle with the reference line.

The computation of the distance DF is simplified if the distance AD is taken as 10 units. For example, if the angle BAC in Fig. 11 is to be $30^\circ 15'$ and the length of AD is to be made equal to 10 units, then the required length of the perpendicular DF is $10 \times \tan 30^\circ 15' = 10 \times 0.58318 = 5.83$ units. Also, the plotting is facilitated if the length of a unit is made equal to one of the main divisions or subdivisions on any convenient edge of the engineers' scale. In the illustration, the unit used in laying off the distance AD is one of the main divisions on the 50-scale, but any other scale could have been used instead. Then, the length of DF is made equal to 5 main divisions and 8.3 subdivisions on the 50-scale. The required number of figures in the computed result depends on the accuracy with which the distance can be laid off and, hence, on the size of the unit assumed in laying off the base AD .

*** 22. Laying off Obtuse Angle by Tangent Method.**—In case it is desired to lay off an obtuse angle by the method of tangents, the reference line from which the angle is to be turned is first produced backwards from the given vertex of the angle. Then, the supplement of the given angle, being an acute angle, is laid off from that prolongation by proceeding in the manner described in the preceding article for an acute angle. For example, an angle of 120° that is turned counter-clockwise from the reference line may be laid off as shown in Fig. 12. The vertex of the angle is at the point A , the reference line is represented by AB , and the required angle BAC is 120° .

First, the reference line AD is produced backwards and the distance AD on this prolongation is made equal to 10 units. In this case, a unit is taken as the length of a subdivision on the 10-scale. The problem now consists in laying off the supplement of the given angle, or $180^\circ - 120^\circ = 60^\circ$, from the line AD . Since the angle of 120° is to be turned counter-clockwise

from AB , the line DE , which is perpendicular to AD at D , is drawn upwards. On this perpendicular the distance DF is made equal to $10 \times \tan 60^\circ = 10 \times 1.7321 = 17.3$ units. The required line AC is then drawn through points A and F .

*** 23. Use of Cotangent.**—Where the angle that a line is to make with an established line is near 90° , the natural tangent is so great that an excessively large right triangle would be required if the method of tangents were applied in the manner previously described. In such a case, the difficulty may be conveniently overcome by constructing a right triangle in which

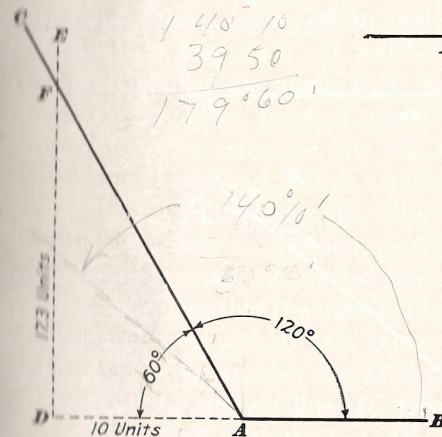


FIG. 12

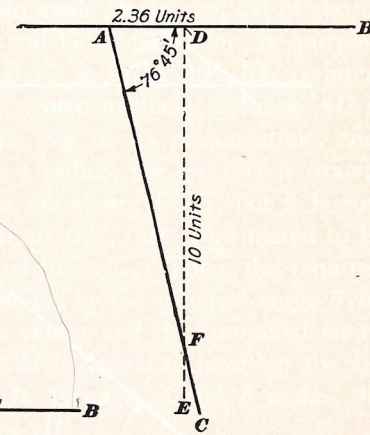


FIG. 13

the length of the side perpendicular to the reference line is made equal to some convenient distance, as 10 units, and the length laid off along the reference line is made equal to the product of the perpendicular side and the cotangent of the required angle. For example, a clockwise angle of $76^\circ 45'$ from a given reference line may be laid off as shown in Fig. 13, where AB is the reference line and AC is the other side of the required angle.

In constructing the right triangle of which the required angle of $76^\circ 45'$ is a part, the length of a unit is taken as one main division on the 60-scale, and the length of the side of the triangle perpendicular to the reference line AB is taken as 10 units. Then, since $\cot 76^\circ 45' = 0.2355$, the first step in plot-

ting the angle is to lay off along AB a distance AD equal to $10 \times 0.2355 = 2.36$ units. Hence, with the zero of the 60-scale at A , the point D is located by laying off a distance equal to two main divisions, three subdivisions, and slightly more than one-half of a fourth subdivision.

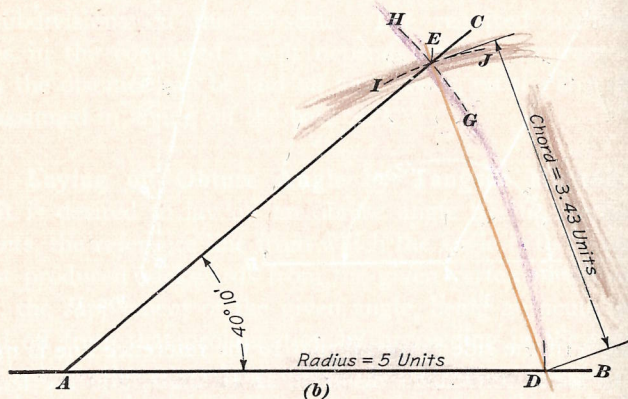
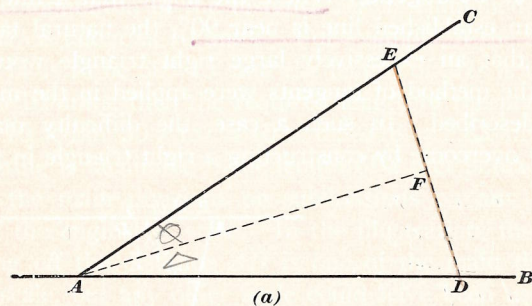


FIG. 14

The perpendicular side of the right triangle is plotted by drawing the line DE perpendicular to AB at D and laying off along this perpendicular the distance DF equal to 10 units, or ten main divisions on the 60-scale. The line AC through A and F then makes an angle of $76^\circ 45'$ with AB .

24. **Laying off Angle by Method of Chords.**—The principle involved in laying off an angle by the method of chords is illustrated in Fig. 14 (*a*), where AB is the reference line and

A is the vertex of the given angle *BAC*. If *AD* and *AE* are equal distances along the sides *AB* and *AC* of the angle, and the line *DE* is drawn, the triangle *ADE* is isosceles. Then, if the line *AF* is the perpendicular bisector of the side *DE*, that line likewise bisects the angle *BAC*. Thus, each of the angles *DAF* and *EAF* equals $\frac{1}{2}$ *BAC*. Also, in the right triangles *DAF* and *EAF*,

$$\text{OPP} = \text{HYP} \times \sin \Delta \quad \& \quad \text{OPP} = \text{HYP} \times \sin \theta$$

$$DF = AD \sin DAF \text{ and } EF = AE \sin EAF$$

Since $DF = EF$, $AD = AE$, and $DAF = EAF = \frac{1}{2} BAC$,
 $\therefore DE = DF + EF = 2 AD \sin \frac{1}{2} BAC$

The procedure in laying off an angle by the method of chords is as shown in view (b). It will be assumed for the purpose of illustration that the angle BAC is to be $40^{\circ} 10'$, counter-clockwise. From the point A , any convenient distance AD is laid off along the reference line AB or its prolongation; and, with A as a center and AD as a radius, the arc GH is drawn. It is not necessary to draw the entire arc DGH , but it is sufficient to draw only the portion, as GH , that is estimated to be long enough to cross the desired line AC . Then, the required length of the chord that is subtended by the specified angle BAC is computed; and, with D as a center and the computed chord length as a radius, another arc IJ is drawn to intersect the arc GH at E . The straight line AC through A and E makes the desired angle BAC with the line AB .

The computation of the length of chord is simplified if the distance AD is made equal to 5 units. Then, for the assumed angle of $40^\circ 10'$, the required length of the chord is

$$\begin{aligned} 2 AD \sin \frac{1}{2} BAC &= 2 \times 5 \times \sin 20^\circ 05' \\ &= 10 \times 0.34339 = 3.43 \text{ units} \end{aligned}$$

In Fig. 14 (b), the length of a unit is taken as one main division on the 20-scale; the radius AD of the arc GH is equal to 5 such divisions; and the radius of the arc whose center is at D has a length of three main divisions and 4.3 subdivisions on the 20-scale.

Some engineering handbooks contain a table giving the lengths of chords for a radius of unit length and various angles.

If such a table is available, the required chord between D and E , Fig. 14, may be found most easily by taking the product of the distance AD and the length of chord obtained from the table. When a table of chords is used, the distance AD should preferably be made equal to 10 units.

PLANNING MAPS

25. Size and General Layout of Maps.—A map that is to be used in the field is generally made on a sheet that has an over-all size between 18 by 24 inches and 24 by 36 inches. In some cases, the entire area to be mapped may be adequately represented on a single sheet; whereas, in other cases, two or more sheets, each showing a part of the area, are required. When more than one sheet is needed to show a map of the entire area, all the sheets of the set should preferably be of the same size.

Regardless of the extent of the area covered by a map, judgment must be exercised in establishing on the original drawing sheet the position of the starting point of the survey and the direction of the meridian with respect to the edge of the sheet, so that all the points in the survey will be plotted on the sheet and the finished map will have a well-balanced appearance. Whenever possible, the map should be so laid out that the direction of the meridian will be parallel to an edge of the sheet. In every case, the general direction of north should be toward the top of the sheet rather than toward the bottom. Where a complete map comprises several sheets, the plotting should be arranged in such a way that the sheets can be fitted to one another so as to form a continuous layout. The sheets should be numbered in regular succession.

26. Essentials in Mapping.—Indispensable requirements for good work in the preparation of a map are accuracy in plotting, clearness in representing and describing the various features, legibility in lettering, and neatness in drawing lines. Also, it is essential that the information given by the map itself should be supplemented by a title that includes the purpose and location of the survey, the name of the surveyor or the corpora-

tion that employed him, the scale of the map, and the date. Another detail that belongs on every map is a line or symbol indicating the direction of the meridian. Usually, this direction is indicated by an arrow. On some maps, it is necessary to give also a key, or legend, which explains the meanings of any special symbols that are used on the map.

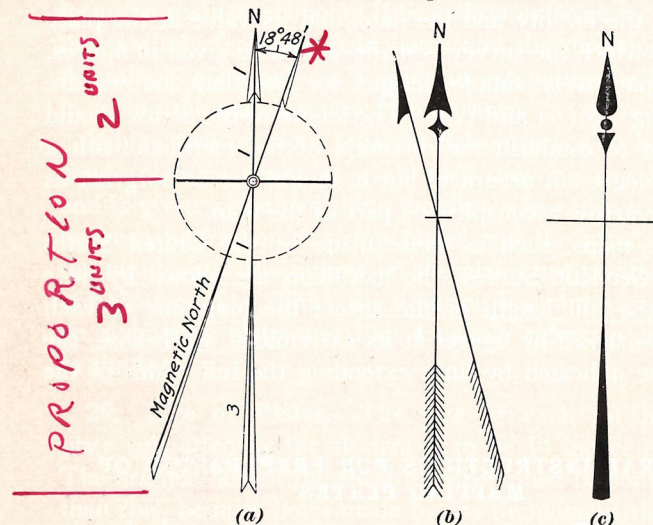


FIG. 15

All lines on a map should be clear and well-defined, as lines that appear faded are a blemish to the map. However, very heavy lines are to be avoided, except when used for shading or for important political boundaries. A neat border line adds greatly to the appearance of a map. It should be wider than the heaviest line on the map.

27. Meridian Arrow.—The meridian arrow on an ordinary map should be simple in design. On some maps, it is sufficient to show the direction of either the true meridian or the magnetic meridian, whereas on other maps both meridians are shown. Usually, the true meridian is represented by an arrow with a full head and also a full tail, if a tail is shown; and only a half-head and a half-tail are used for the magnetic meridian. In Fig. 15 (a) is illustrated a good method of showing both the

true and magnetic meridians, and such meridian arrows are used on the plates for this text. The dotted circle and the numbers 1 and 3 would not appear on the map, but they are here given in order to indicate the approximate proportions to be used in constructing well-balanced arrows. The angle between the true and magnetic meridians, as $18^{\circ} 48'$ in the illustration, varies with the locality and the date, and its value at the place and time at which the survey is made should be given in figures. Other comparatively simple designs for meridian arrows are shown in Fig. 15 (b) and (c). The length of the arrow should be sufficient to establish the direction of the meridian with a reasonable degree of accuracy, but it should not be so great as to make the arrow a conspicuous part of the map.

On some maps, such as those made by the United States Coast and Geodetic Survey, the meridians are shown by lines extending the full length of the sheet; the longitude of each such line is marked by figures at its extremities. Likewise, the latitudes are indicated by lines extending the full width of the sheet.

GENERAL INSTRUCTIONS FOR PREPARATION OF MAPPING PLATES

28. General Remarks.—In connection with each drawing plate to be made by the student, there is given a preliminary explanation covering the following features: What is to be shown on the plate; the scale or scales to be used; the direction of the meridian with respect to an edge of the drawing paper; and the location of a sufficient number of controlling points to fix the general layout of the plate. This information is supplemented by a small-size sample map, which is included in the text to serve as a general guide to the student in preparing his drawing. Each controlling point should be located in approximately the same relative position on the student's plate as on the sample map.

After the general layout of the map has been established, the plotting of the map is carried out in accordance with the field notes given in the text. In addition, the text includes detailed directions for executing the work. These directions are given

not only for the purpose of enabling the student to prepare the particular plate to which they refer but also to familiarize him with good practice. They should therefore be studied carefully, so that the plotting may be done with an understanding of the principles involved rather than as a mere mechanical operation. Obviously, the student will derive the most benefit from the texts on mapping if he endeavors to carry out the plotting entirely from the field notes and the directions, and uses the sample plate only as a guide in arranging the lettering and in giving the map a finished appearance.

Where two or more separate drawings are to be made on a single plate in this text, each such drawing is called a *figure* and is given a distinguishing number. In order to differentiate between figures on drawing plates and the usual illustrations, the numbers of the figures on the plates are printed in the text in bold-face type. Thus, the notation **Fig. 4** designates a figure on a drawing plate, and Fig. 4 merely refers to an illustration in the text.

29. Size of Plates.—The size of each finished drawing plate submitted by the student is to be 14 inches by 18 inches. The original size of the sheet of paper must be somewhat larger than this, because holes made by the thumbtacks that fasten the paper to the drawing board should not appear on the finished plate. Extra space outside the limits of the plate is also convenient for testing the ruling pen in order to make sure that the ink flows freely and the lines have the proper width.

On the finished plate, there is to be a border line that is $\frac{1}{2}$ inch from each edge. Thus, the plotting is to be confined to a rectangular space 13 in. \times 17 in.

30. Lettering on Maps.—A map is apt to be judged by the quality of the lettering on it. Nothing adds more to the finished appearance of the map than good lettering, whereas poor and slovenly lettering will reduce considerably the value of an otherwise perfect drawing. Legibility and uniformity are the chief requirements of good lettering. Except for the titles of very elaborate maps, ornamental letters are entirely out of place. In general, the lettering should be so arranged that the letters

** WE WILL USE NORMAL DRAFTING STYLE.*

will appear right side up when the map is held in its normal position. Where a line of lettering is other than horizontal, it should ordinarily run so that the user of the map will read either upwards or downwards toward the right. If the line of lettering is vertical, the lettering should read upwards.

The size of letters must be governed by the size of the map and by the character of the title, name, or statement to be shown. The lettering should be neither so prominent that it would obscure important details of the map, nor so small that it could not be read easily.

For best results in lettering, the lines should be bold and sharp. Lines that are too fine detract from, rather than add to, the appearance of the map. The fault of attempting to make very fine lines is common with beginners and should be avoided. All dimensions should be expressed by means of figures. Also, descriptions of important lines and objects should be as brief as possible without being ambiguous or inaccurate.

All the letters of a unit, such as a name, and also the letters of different units of similar character should be of uniform size. Light pencil guide lines for the bottoms and tops of the letters should be drawn as an aid in making the letters of uniform height. Uniformity in the spacing and slant of the letters is as important as uniformity in size. The letters of a word should be so spaced that the black and white areas will be well balanced.

The position of the title on the sheet will depend largely on the shape of the tract of ground covered by the map. The lettering in the title should harmonize in character and size with the rest of the lettering on the map.

31. Formation of Roman Letters.—Capital letters of the style shown in Fig. 16, which are known as upright modern Roman capitals, are very popular for titles on maps. Such letters, $\frac{3}{16}$ inch high, will be used in the titles on all three drawing plates in this text on mapping. Skill in lettering can be acquired only by practice. One who has had insufficient practice in making Roman letters should therefore become familiar with their construction before he attempts to put the title on the first drawing plate. A good method for learning to form these

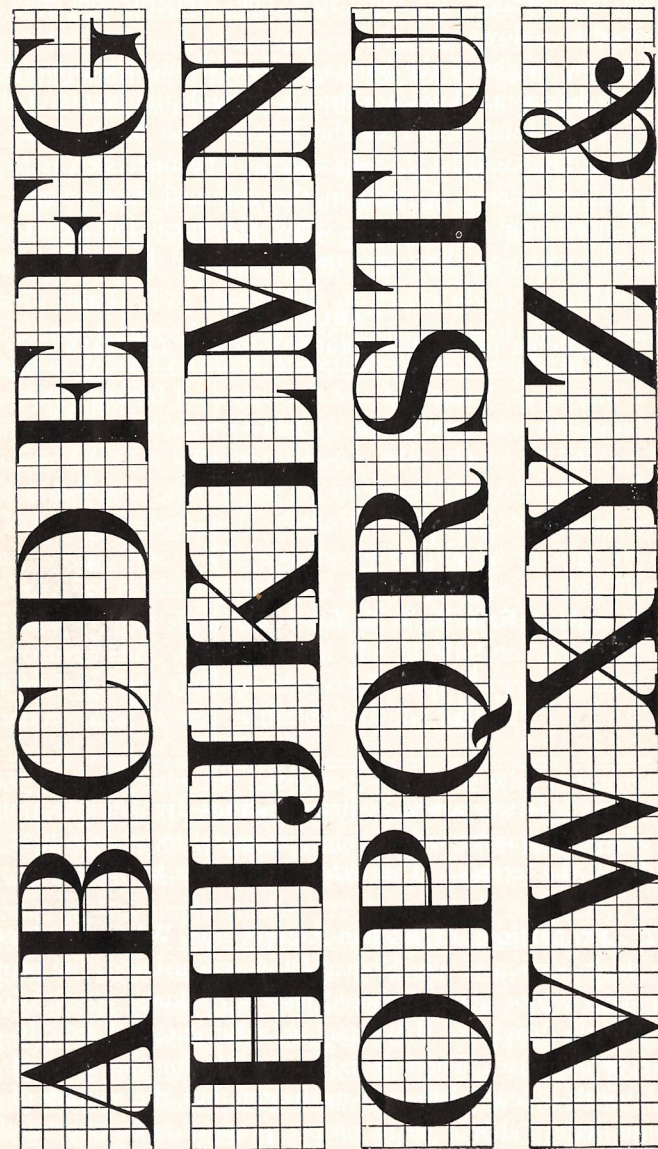


Fig. 16

letters is indicated in Fig. 16. The construction may be described as follows:

First, the height of the letters is divided into six equal parts and light horizontal lines are drawn through the points of division. If the total height of the letters is to be $\frac{3}{10}$ inch, the horizontal lines will be $\frac{1}{20}$ inch apart. The next step is to lay off equal distances— $\frac{1}{20}$ inch in the assumed case—along the lowest horizontal line, and to draw a vertical line from each point so located to the uppermost horizontal line. The space

TABLE I
WIDTHS OF UPRIGHT MODERN ROMAN CAPITALS

Letter	Width, in Sixths of Height
I	1
J	$3\frac{3}{4}$
N	4
F, L, U	$4\frac{1}{4}$
E, P, R	$4\frac{1}{2}$
B, S, Z	$4\frac{3}{4}$
A, D, H, K, M, V, X, Y	5
C, G	$5\frac{1}{4}$
T, &	$5\frac{1}{2}$
O, Q	6
W	7

in which the letters are to be drawn is thus divided into a number of small squares whose sides are equal to one-sixth of the height of the letters. The letters may then be outlined.

32. Proportions of Roman Letters and Width of Spaces.

The thick portion of any Roman letter, exclusive of the projection at its top or bottom, should have a thickness equal to one-sixth of the height of the letter, or a thickness of exactly one space in Fig. 16. There is no rigid rule for the ratio of the total width of each letter to its height, but the widths of the various letters should be in proper proportion to one another. Good results are obtained if the widths, not including the small projections, are as listed in Table I. These widths are used in

Fig. 16 and should be used also in the titles of all drawing plates that are made in connection with these texts. The corresponding width of a projection at the top or bottom of a letter should be almost equal to the thickness of the thick portion of the letter.

The proper spacing of the letters is extremely important. Because of the great number of possible combinations of pairs of letters, no specific values are given for the distances between particular letters. However, the following principles should be kept in mind. For the widths of letters given in Table I, the horizontal distance between letters, exclusive of the projections, should in general be about one-quarter of the height of the letter. The space should be somewhat less between *C* or *G* and a preceding letter, between *O* and *Q* and a letter on either side, or between *D* and a following letter. Also, where *A* follows *P* or is on either side of *T*, *V*, or *W*, or where *L* is followed by *T*, *V*, or *W*, the adjacent ends of the letters should be about in the same vertical line. The spacing of the letters in Fig. 16 does not conform to these principles, as the purpose of this illustration is merely to show the details of construction of the letters. For convenience, the letters in the illustration are spaced so that each upright thick portion comes between two vertical construction lines, except in the case of the letter *F*.

33. Suggestions for Lettering.—Anyone who is not expert in lettering should rule the straight parts of the various letters. Also, he should draw these straight lines before the curves are drawn. Each curve should first be formed as accurately as possible with a single stroke. If the result is reasonably satisfactory, the curve should not be changed, as altering the letter slightly is more likely to make it worse than to improve it. When ink is being used in lettering, a ruling pen should be employed for making the straight lines. A uniform thickness can then be readily maintained for all the thin straight portions of the letters. Each thickened straight portion should be made by first ruling two lines along the edges, and afterwards filling in the space between those lines, so that the letters appear as in Fig. 16.

The recommended procedure in putting the title on a drawing in ink is as follows: First, horizontal and vertical construction lines, such as those shown in Fig. 16, should be drawn lightly in pencil. Then, the outlines of the letters should be carefully sketched in pencil. Finally, the letters should be inked in, and the construction lines should be erased.

34. **Directions for Sending in Plates.**—As soon as the student completes each plate, he should send it to the Schools for examination. The plates should preferably be submitted one at a time. They are returned to the student with any corrections that may be necessary and with suggestions that will aid him in subsequent work. In a 1 in. \times 3 in. rectangular space in the lower right-hand corner of each plate in *Mapping*, Part 1, the student should letter in single-stroke vertical lettering the number of the plate, the date, his name, and his class letters and number, as shown in Fig. 17 and as indicated on the sample

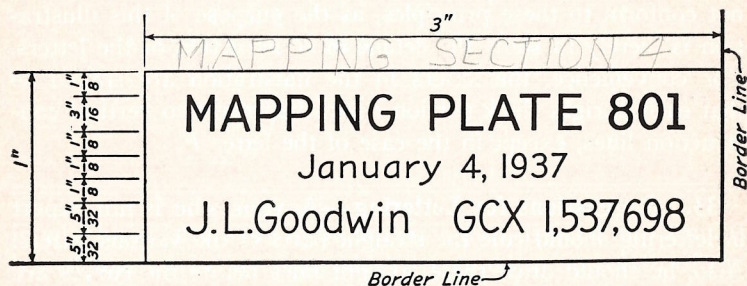


FIG. 17

plates. Also, it is very important that the student write his name and address in full in lead pencil on the back of the plate.

Each of the three drawing plates in Mapping, Part 1, should be made in pencil on drawing paper. The main lines and other information that would appear on a finished map must be made fairly heavy. The construction lines should be made lightly, and should not be erased. No inking in is required.

1 CHAIN = 100 LINKS

PLOTTING TRAVERSES

LOCATION OF POINTS ON TRAVERSES

35. Traverses and Courses.—A traverse is a series of connected lines, the directions and lengths of which are established by means of a survey. Each line of a traverse is called a course. If a traverse begins and ends at the same point, thus forming a closed figure, the traverse is said to be closed. For example, the boundaries of a tract of land form a closed traverse. On the other hand, if the end of the last course of the traverse does not coincide with the beginning of the first course, the traverse is open. A typical example of an open traverse is the preliminary line for the location of a railroad or highway.

36. Methods of Indicating Lengths of Courses.—The lengths of the courses of a traverse are usually measured in feet and decimal parts of a foot. However, in some land surveys, distances along boundaries are measured in chains and links, a chain being equivalent to 66 feet and containing 100 links.

In the case of a closed traverse, it is customary to give the numerical value for the length of each course and to identify the points at the intersections of two courses by means of letters, numbers, or names. However, in the case of an open traverse, it is usually desirable to know at a glance the total length of the traverse from the point of beginning to any particular point. A point on an open traverse is therefore commonly identified by the word station and a number expressing the distance from the beginning of the traverse to the point. The starting point of the traverse is station zero, usually written Sta. 0 + 00, or sometimes simply Sta. 0. Points at 100-foot intervals from the point of beginning are known as *full stations*. Thus, a point 600 feet from the starting point is Sta. 6 + 00, or Sta. 6. Where the distance from Sta. 0 + 00 to a certain point is not a multiple of 100 feet, that point is called a *plus station*. The

distance from Sta. 0 + 00 to a plus station is expressed by the number of the full 100-foot station immediately preceding the plus station and the distance from that full station to the plus station. Thus, a point at a distance of 679.32 feet from Sta. 0 + 00 is called Sta. 6 + 79.32.

Obviously, the length of any course of an open traverse is equal to the difference between the station numbers of the extremities of the course. For instance, the distance between Sta. 8 + 25 and Sta. 11 + 14 is $1,114 - 825 = 289$ feet.

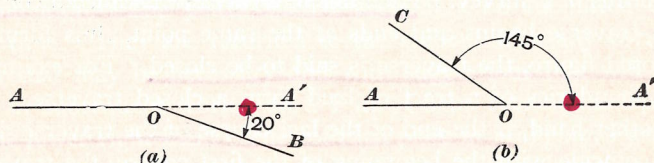


FIG. 18

37. Methods of Indicating Directions of Courses.—The direction of a course of a traverse is usually given by its bearing or azimuth, either of which is the angle that the course makes with the meridian. *However, in many open traverses, the direction of each course after the first one is established by the angle that it makes with the preceding course. This angle may be measured either from the preceding course itself or from the prolongation of the preceding course.

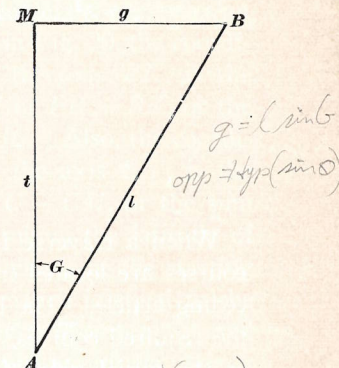
An angle between a course and the prolongation of the preceding course is known as a deflection angle. A deflection angle is recorded as being to the right or to the left, according as the angle is turned clockwise or counter-clockwise from the prolongation of the reference course. For instance, in Fig. 18 (a), the deflection angle between the prolongation OA' of the course AO and the line OB is 20° to the right, or 20° R.; and in view (b), the deflection angle from the prolongation OA' of the course OA to the line OC is 145° L.

38. Plotting Traverse by Directions and Lengths of Courses.—The relative positions of two points may be established by the direction and length of the straight line between the points. Thus, the successive courses of a traverse can be

plotted as follows: Through a plotted point that represents the beginning of the traverse, a line is drawn in the direction of the first course, and the length of that course is laid off along the line. Obviously, the end of the measurement is the beginning of the second course of the traverse. That course is then plotted by drawing through its point of beginning a line making the proper angle with the meridian or with the first course, and the length of the second course is laid off along this line. Each of the other courses is plotted in succession in a similar manner. The traverse $ABCD$, Fig. 10, was plotted by this method.

39. Plotting by ^(N-S) Latitudes and ^(E-W) Departures of Courses. Another common method of plotting a traverse consists in locating

each course by constructing a right triangle. The method is somewhat similar to that described in Art. 21 for laying off an angle by its tangent. However, in this method, the length of the hypotenuse of the right triangle is made equal to the length of the course to be plotted. For instance, if it is desired to plot a course AB , Fig. 19, that has a length l and makes an angle G with the reference line AM , the sides AM and MB of the right triangle AMB are laid off. Obviously, the



lengths t and g of the perpendicular sides of the triangle are computed by the relations $t = l \cos G$ and $g = l \sin G$. When this method of plotting a traverse is adopted, the perpendicular sides of the right triangles for all the courses are usually made parallel and perpendicular to the meridian. For a particular course, the side parallel to the meridian is called the latitude of the course and the side perpendicular to the meridian is the departure of the course.

The latitude of any course is equal to the product of the length of the course and the cosine of the angle that the course makes with the meridian. Also, the departure of the course is equal to the product of its length and the sine of that angle. Whether the latitude should be laid off upward or downward and whether

the departure should be laid off to the right or to the left depends on the bearing or azimuth of the course and can be readily determined by the letters in the bearing or by the value of the azimuth. Thus, if the bearing is northeast, the latitude is laid off toward the north and the departure toward the east. If the azimuth measured from the north is between 180° and 270° , as in the case of the line OB , Fig. 4, the latitude is laid off toward the south and the departure toward the west.

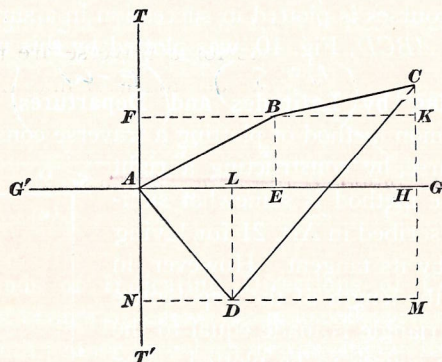


FIG. 20

When a traverse is plotted by latitudes and departures, the courses are located in succession, as in the method of the preceding article. The hypotenuse of each right triangle is one of the required courses. The triangle for the first course is begun at the initial point of the traverse, and the triangle for each other course is begun at the end of the preceding course. It is advisable to check the plotting by measuring the plotted length of each course and comparing the scaled distance with the length given in the field notes.

40. Plotting by Coordinates.—In the methods described in the preceding two articles, the plotted position of each course depends on the positions of the preceding courses, and any error in one course is carried to all courses that follow. Another method of plotting traverses consists in locating the end of each course by the perpendicular distances to that point from each of two reference lines which are at right angles to each other. These two distances for a given point are known as the rectan-

gular coordinates, or simply coordinates, of the point; and the reference lines are called coordinate axes. For instance, in Fig. 20, the lines TT' and GG' represent coordinate axes, and the corners A , B , C , and D of the closed traverse $ABCD$ may be located by their coordinates with respect to those axes. The coordinates of the point A are each zero; those of B are either the distances AE and EB or the distances AF and FB ; those of C are the distances AH and HC ; and those of D are either the distances AL and LD or the distances AN and ND .

Usually, the coordinate axes for a traverse are the meridian and the parallel of latitude, or east-and-west line, through some point on the earth's surface. When the coordinates of any point on a traverse are known, the coordinates of all other points can be readily computed from the latitudes and departures of the courses of the traverse. For instance, in Fig. 20, the coordinate of the corner C , as measured from the axis TT' , is the distance AH , which is equal to $AE + EH$ or $FB + BK$, or the sum of the departures of courses AB and BC . Also, the coordinate of the corner D , as measured from the axis GG' , is the distance HM , which is equal to $HK + KC - CM$, or the sum of the latitudes of courses AB and BC minus the latitude of course CD . Since DA is the course that closes at the starting point, the coordinate is the distance LD , which is equal to the latitude of AD . After the ends of the courses have been plotted by coordinates, the courses are drawn between the proper points.

Plotting points by coordinates has the advantage that the plotted position of each point is independent of the positions of other points. Hence, an error in the location of one point does not affect any other point that is plotted by coordinates. When this method is used, the work is conveniently checked by scaling the lengths of the courses.

MAPPING PLATE 801: PLOTTING OPEN TRAVERSES

41. Description of Plate.—In Plate 801, six open traverses are to be plotted, each by a different method. The field notes containing the data for plotting the various courses are given in Tables II to VII, inclusive. Where the distances for locating

the survey points are expressed in stations of 100 feet, as is the case in Tables II, III, IV, V, and VII and as is usually the case in practice, the traverse is to be plotted to a scale of 200 feet to the inch. The lengths of the courses in Table VI are given in chains, and that traverse is to be plotted to a scale of 2 chains to the inch.

The meridian for all traverses is here assumed to be parallel to the longer pair of opposite border lines of the plate. Also, with the plate turned so that its longer border lines are vertical, the starting point of each traverse is located at a distance of $\frac{3}{4}$ inch from the left-hand border line. Therefore, instead of drawing a separate line to represent the meridian through the starting point in each figure, it is more convenient to draw a continuous construction line for the entire length of the plate. This line should obviously be parallel to the left-hand border line and $\frac{3}{4}$ inch from it.

TABLE II
NOTES FOR TRAVERSE IN FIG. 1 OF PLATE 801

Station	Deflection Angle	Bearing
29 + 18	End of line	END OF LINE
23 + 98	102° 42' L.	N 21° 24' E
17 + 26	66° 21' R.	S 55° 54' E
12 + 62	37° 45' L.	N 57° 45' E
5 + 25	34° 30' L.	S 84° 30' E
0 + 00		S 50° 00' E

In practical work, construction lines used in plotting angles are drawn lightly in pencil for temporary reference. Such lines may be left on the drawing or may be erased as soon as the angles have been laid off, depending on the intended use of the original drawing. In this plate, the construction lines should be left on the drawing in order to indicate the method of procedure followed by the student. On the sample plate, these construction lines are indicated by dotted lines, and the courses of the traverses are shown by full lines. The student should show the construction lines by light full lines and show the

traverse courses fairly heavy. Each angle point of a traverse should be marked by a needle hole with a small circle around it. Construction points should also be marked by needle holes but such points should not be circled on the finished drawing.

Values of various angles and distances that would not appear on a map in practice are included in parentheses on the sample plate for the purpose of making the explanations clearer to the student. These values and also the reference letters should be omitted from the plate prepared by the student.

42. Notes and Instructions for Plotting Deflection Angles by Semicircular Protractor.—The notes for the traverse that is to be plotted in Fig. 1 are given in Table II. The stations designated in the first column are the vertexes of the deflection angles whose values are listed in the second column. As indicated by the direction in which the station numbers increase, the notes were recorded in the field from the bottom of the page upward, this being a common practice. In the third column is shown the bearing of the first course from Sta. 0 + 00 to Sta. 5 + 25.

In plotting the traverse in Fig. 1 on the plate, all angles are to be laid off with an ordinary semicircular protractor. The starting point *A*, or Sta. 0 + 00, is to be arbitrarily located at an actual distance of $\frac{3}{4}$ inch from the upper border line. Since the bearing of the first course from Sta. 0 + 00 to Sta. 5 + 25 is S 50° E, the course lies to the right of the meridian through Sta. 0 + 00 and makes an angle of 50° with the south half of that meridian. Hence, to establish the direction of the course, the first step is to place the protractor with its center at *A*, its zero mark in the meridian, and its graduated semicircle to the right of the meridian. Then a point opposite the 50° graduation in the southerly quadrant is marked on the plate, and a straight line is drawn through *A* and this point. That line has a bearing of S 50° E.

The course in Fig. 1 from Sta. 0 + 00 to Sta. 5 + 25 obviously has a length of 525 feet. Hence, Sta. 5 + 25, or point *B*, is located on the line having the direction of that course by measuring from Sta. 0 + 00, or point *A*, a distance of 525 feet to

the specified scale of 1 inch = 200 feet. For laying off distances to this scale, it is most convenient to use the 20-scale of the engineers' scale. Then, each main division represents 100 feet and each subdivision represents 10 feet. Distances can be laid off quite accurately to the nearest foot by estimating tenths of a subdivision.

43. In order that the deflection angle at point *B*, Fig. 1, may be laid off by means of a protractor, the length of the line establishing the direction of the course *AB* should at first be made considerably more than 525 feet. The actual length of the prolongation beyond *B* should be a little greater than the radius of the protractor. The prolongations shown in Fig. 1 on the sample plate are merely long enough to indicate the lines from which the deflection angles are measured and are not intended to be so long as the radius of an ordinary protractor.

The deflection angle at Sta. 5 + 25 is $34^{\circ} 30'$ to the left. To lay off this angle, and thus establish the direction of the course from Sta. 5 + 25 to Sta. 12 + 62, the protractor is placed as accurately as possible with its center at the point *B*, with its zero point on the prolongation of the course *AB*, and with its graduated semicircle to the left of that prolongation or toward the top border line. Then, a point opposite $34^{\circ} 30'$ on the graduated edge of the protractor is marked on the plate, and a straight line of unlimited length is drawn through *B* and this point. The required course lies on this line, and the point *C*, or Sta. 12 + 62, is located by measuring along the line from *B* a distance of $1,262 - 525 = 737$ feet, to the specified scale of 1 inch = 200 feet.

The remaining angles and courses are plotted in a similar manner. To determine the direction of the course *CD* from Sta. 12 + 62 to Sta. 17 + 26, the protractor is first placed with its center at *C*, its zero point on the prolongation of *BC*, and its graduated semicircle to the left of that prolongation. The point opposite $37^{\circ} 45'$ on the graduated arc is then marked and a line is drawn through *C* and that point. Point *D*, or Sta. 17 + 26, is located along that line at a distance of $1,726 - 1,262 = 464$ feet, to scale, from *C*.

At *D*, an angle of $66^{\circ} 21'$ is laid off to the right from the prolongation of course *CD*. On the line thus established, the point *E*, or Sta. 23 + 98, is located at a distance of $2,398 - 1,726 = 672$ feet from *D*. Finally, the angle of $102^{\circ} 42'$ at *E* is laid off to the left of the prolongation of the course *DE*; and the end *F* of the traverse, or Sta. 29 + 18, is located on the line so determined at a distance of $2,918 - 2,398 = 520$ feet from *E*.

44. **Plotting Bearings by Protractor.**—The direction of each course of the traverse in Fig. 2 of Plate 801 is to be determined by laying off the bearing of the course with a protractor.

The notes that are needed for the plotting are given in Table III. In the first column are the station numbers of the points at which the various courses of the traverse begin; and in the second column are the bearings of the respective courses. The bearing of a course is listed opposite the station of its beginning point. Thus, the bearing of the course

TABLE III
NOTES FOR TRAVERSE IN FIG. 2
OF PLATE 801

Station	Bearing
25 + 20	End of line
20 + 60	S $45^{\circ} 00'$ E
15 + 97	S $70^{\circ} 45'$ E
11 + 25	N $80^{\circ} 30'$ E
7 + 45	S $81^{\circ} 20'$ E
3 + 95	N $83^{\circ} 30'$ E
0 + 00	N $60^{\circ} 00'$ E

from Sta. 3 + 95 to Sta. 7 + 45 is tabulated opposite the former station as N $83^{\circ} 30'$ E.

The starting point *A*, or Sta. 0 + 00, of the traverse should be located at an actual distance of $5\frac{1}{2}$ inches from the upper border line. In order that the bearing of a course may be laid off with the protractor, a fairly long construction line that is parallel to the meridian must be extended in both directions from the beginning of the course. Since the meridian is parallel to a border line, all lines parallel to the meridian can be conveniently drawn with a T square and a triangle. First, the T square is placed in such a position that the upper edge of the blade is a few inches below the plotted point through which the line having the direction of the meridian is to

be drawn. Then, the short edge of the triangle is set in contact with the upper edge of the blade of the T square, and the triangle is slid along until its perpendicular edge passes through the plotted point. Finally, the line representing the meridian is drawn along that edge of the triangle.

45. In plotting the traverse in Fig. 2, the direction of the first course AB from Sta. $0 + 00$ to Sta. $3 + 95$ is established in the following manner: Since the bearing of the course is $N 60^\circ 00' E$, the protractor is set so that its center is at A , its zero mark is on the meridian through A , and its graduated semicircle is to the east, or right, of that meridian. Then, a point in the northeast quadrant and opposite $60^\circ 00'$ on the graduated edge is marked on the plate, and a line is drawn from A through that point. The length of this line should be somewhat greater than the length of the first course, or 395 feet, to the specified scale of 1 inch = 200 feet; and the point B , or Sta. $3 + 95$, is located along the line at a distance of 395 feet from A .

The next course, which extends from Sta. $3 + 95$ to Sta. $7 + 45$, may be plotted by proceeding as follows: A line having the direction of the meridian is drawn through point B ; and, since the bearing of the new course is $N 83^\circ 30' E$, the protractor is set with its center at B , its zero mark in the meridian through B , and its graduated semicircle to the right of B . Then, a point in the northeast quadrant opposite $83^\circ 30'$ on the graduated edge is marked on the plate, a line is drawn from B through that point, and the point C at Sta. $7 + 45$ is located on this line at a distance of $745 - 395 = 350$ feet from B .

Since the bearing of the course CD from Sta. $7 + 45$ to Sta. $11 + 25$ is $S 81^\circ 20' E$, the angle $81^\circ 20'$ is laid off counter-clockwise from the south portion of the meridian line through point C , but otherwise the procedure for plotting that course is similar to that described for the first two courses of the traverse. The length of the course is $1,125 - 745 = 380$ feet. Likewise, each of the remaining courses is plotted by following the same general method of procedure. In each case, the protractor is placed to the right of the meridian.

The course from Sta. $11 + 25$ to Sta. $15 + 97$ has a length of 472 feet and makes an angle of $80^\circ 30'$ clockwise with the north portion of the meridian at Sta. $11 + 25$. The length of the course from Sta. $15 + 97$ to Sta. $20 + 60$ is 463 feet and it makes an angle of $70^\circ 45'$ counter-clockwise with the south portion of the meridian at Sta. $15 + 97$. For the final course, the length is $2,520 - 2,060 = 460$ feet and the angle with the south portion of the meridian at Sta. $20 + 60$ is $45^\circ 00'$, counter-clockwise.

TABLE IV
NOTES FOR TRAVERSE IN FIG. 3 OF PLATE 801

Station	Deflection Angle	Bearing
24 + 00	End of line	
18 + 00	$27^\circ 40' L.$	
13 + 91	$21^\circ 15' R.$	
6 + 43	$44^\circ 00' R.$	
0 + 00		$N 53^\circ 30' E$

46. **Plotting Deflection Angles by Method of Tangents.** The notes for the traverse to be plotted in Fig. 3 are given in Table IV. They are similar to those in Table II, but in this case the deflection angles are to be laid off by the method of tangents. Since the 20-scale is used for measuring the lengths of the courses, it is preferable to use the same scale for laying off the right triangle for plotting each angle. In this figure, a convenient length of a unit is one-half of a main division, or 5 subdivisions, on the 20-scale. Hence, if the side of the triangle in the prolongation of the reference line is given a length of 10 units, its actual length will be equal to 5 main divisions on the 20-scale.

Sta. $0 + 00$, or point A , is located at an actual distance of $8\frac{1}{4}$ inches from the upper border line. Also, the first course from Sta. $0 + 00$ to Sta. $6 + 43$ has a bearing of $N 53^\circ 30' E$, and it may be assumed that the angle between the first course and the meridian is a deflection angle to the right of the meridian. Therefore, a distance AB of 10 units on the recommended

scale is laid off from point *A* northward along the meridian, and a line that is perpendicular to the meridian is drawn toward the right at point *B*. Along this perpendicular is laid off a distance *BC* equal to $10 \times \tan 53^\circ 30' = 10 \times 1.351 = 13.51$ units. Since each unit is represented by one-half of a main division on the 20-scale, the distance of 13.51 units is laid off by setting the zero mark of the scale at *B* and counting $13.51 \div 2 = 6.755$ main divisions, or six main divisions and seven and one-half subdivisions. Then a straight line is drawn through *A* and *C* and Sta. 6 + 43 is located at point *D* on that line by measuring from *A* a distance of 643 feet to the specified scale of 1 inch = 200 feet.

The course from Sta. 6 + 43 to Sta. 13 + 91, which has a length of $1,391 - 643 = 748$ feet and makes a deflection angle of $44^\circ 00'$ to the right with the preceding course, is plotted in a similar manner. On the prolongation of course *AD*, a distance *DE* of 10 units is laid off; and, at the end of that measurement, a perpendicular is drawn toward the right. Along this perpendicular is laid off a distance *EF* equal to $10 \times \tan 44^\circ 00' = 9.66$ units. Then, a straight line is drawn through points *D* and *F*, and Sta. 13 + 91 is located on that line at a distance of 748 feet, to scale, from *D*.

The remaining two courses of the traverse from Sta. 13 + 91 to Sta. 18 + 00 and from Sta. 18 + 00 to Sta. 24 + 00 are plotted similarly. The length of the former course is $1,800 - 1,391 = 409$ feet, and the length of the side of the right triangle opposite the required deflection angle of $21^\circ 15'$ should be $10 \times \tan 21^\circ 15' = 3.89$ units. Also, the distance from Sta. 18 + 00 to Sta. 24 + 00 is 600 feet, and the required length of the side of the right triangle opposite the deflection angle at Sta. 18 + 00 is $10 \times \tan 27^\circ 40' = 5.24$ units.

47. Plotting Bearings by Methods of Tangents and Cotangents.—The traverse in Fig. 4 is to be plotted from the notes in Table V, which are similar to those in Table III. The directions of the courses of this traverse are to be established by laying off each bearing angle by its tangent or cotangent. It is therefore necessary to construct for each course a right tri-

angle whose perpendicular sides are, respectively, parallel and at right angles to the meridian. In the construction of these triangles, the length of a unit should be the same as in Fig. 3, and the length of one of the perpendicular sides should be made equal to 10 units. If the angle is laid off by its tangent, the side parallel to the meridian will have a length of 10 units; and, if the angle is so near 90° that the cotangent is more convenient, the side perpendicular to the meridian will be 10 units long. The starting point *A*, or Sta. 0 + 00, of the traverse is located at an actual distance of 11 inches from the upper border line.

48. The bearing of the course from Sta. 0 + 00 to Sta. 4 + 22 in Fig. 4 is $N 40^\circ 30' E$ and that course is plotted in the

TABLE V
NOTES FOR TRAVERSE IN FIG. 4
OF PLATE 801

Station	Bearing
32 + 27	End of Line
26 + 27	N $81^\circ 45' W$
19 + 67	S $45^\circ 15' E$
10 + 97	S $75^\circ 30' E$
4 + 22	N $65^\circ 15' E$
0 + 00	N $40^\circ 30' E$

following manner: Along a line representing the meridian at Sta. 0 + 00, the distance *AB* toward the north is made equal to 10 units, or 5 main divisions on the 20-scale. Then, a perpendicular to the meridian is drawn through *B* to the right, or eastward, and a distance *BC* equal to $10 \times \tan 40^\circ 30' = 8.54$ units is measured

along that perpendicular. The course from Sta. 0 + 00 to Sta. 4 + 22 lies on the line through *A* and *C*; and the point *D*, or Sta. 4 + 22, is located by laying off from *A* a distance of 422 feet, to the specified scale of 200 feet to the inch.

Since the bearing of the course from Sta. 4 + 22 to Sta. 10 + 97 is $N 65^\circ 15' E$, it is convenient to use the cotangent for plotting the angle between the meridian and that course. The distance *DE* to be laid off toward the north along a line representing the meridian at *D* is $10 \times \cot 65^\circ 15' = 4.61$ units, and the distance *EF* to be laid off toward the east along the perpendicular at *E* is 10 units. The point *G*, or Sta. 10 + 97, is then located by laying off, along the line through *D* and *F*, a distance *DG* of $1,097 - 422 = 675$ feet.

The direction of the next course, from Sta. 10 + 97 to Sta. 19 + 67, is likewise established by the method of cotangents. The bearing being S 75° 30' E, the meridian through *G* is drawn toward the south and the distance *GH* along it is made equal to $10 \times \cot 75^\circ 30' = 2.59$ units. Then, a perpendicular to *GH* is drawn through *H* toward the east, a distance *HI* of 10 units is laid off along that perpendicular, and the line through points *G* and *I* is prolonged to Sta. 19 + 67, which is at a distance of 870 feet from *G*.

TABLE VI
NOTES FOR TRAVERSE IN FIG. 5 OF PLATE 801

Course	Bearing	Length Chains	Latitude Chains	Departure Chains
5-6	S 30° 45' W	5.53	- 4.75	- 2.83
4-5	S 70° 15' E	7.77	- 2.63	+ 7.31
3-4	N 25° 30' E	4.62	+ 4.17	+ 1.99
2-3	S 65° 30' E	5.25	- 2.18	+ 4.78
1-2	N 45° 00' E	6.00	+ 4.24	+ 4.24

The course from Sta. 19 + 67 to Sta. 26 + 27 has a bearing of S 45° 15' E and a length of 660 feet. The angle 45° 15' is laid off by the method of tangents by measuring 10 units to the south along a line representing the meridian at Sta. 19 + 67, constructing a perpendicular to the meridian at the end of that measurement, and laying off $10 \times \tan 45^\circ 15' = 10.09$ units on that perpendicular. Sta. 26 + 27 is then located, on the hypotenuse of the right triangle thus laid out, at a distance of 660 feet from Sta. 19 + 67.

In the case of the final course of the traverse, the bearing is N 81° 45' W, and the method of cotangents is again employed. The required length of the side of the right triangle that is in the meridian at Sta. 26 + 27 is $10 \times \cot 81^\circ 45' = 1.45$ units, and the length of the perpendicular side is 10 units. Obviously, the distance of 1.45 units is laid off upward from Sta. 26 + 27 and the distance of 10 units is laid off to the west, or left, of the meridian. The end of the traverse, or Sta. 32 + 27, is located

on the hypotenuse of the right triangle at a distance of $3,227 - 2,627 = 600$ feet from Sta. 26 + 27.

49. Plotting by Latitudes and Departures of Courses.

The notes from which the traverse in Fig. 5 is plotted are given in Table VI. The measurements that are made in the field for each course are the bearing, which is given in the second column, and the length, given in the third column. This traverse represents a part of a closed traverse for a boundary survey. Therefore, the corners are identified by consecutive cardinal numbers, and not by station numbers; and the lengths of the courses are expressed in chains and are given directly in the notes.

In this case, each course is to be plotted by the method of latitudes and departures discussed in Art. 39. Hence, it is necessary to know the latitude and departure of each course. These values, which may be computed as explained in Art. 39, are given in the last two columns of Table VI. Thus, since the bearing of the course 1-2 is N 45° 00' E and its length is 6.00 chains, the latitude is $6 \cos 45^\circ = 6 \times 0.70711 = 4.24$ chains and the departure is $6 \sin 45^\circ = 4.24$ chains. Positive latitudes are laid off toward the north, or upward; and positive departures are laid off toward the east, or to the right. Negative values are laid off in the opposite directions. For laying off distances to the specified scale of 1 inch = 2 chains, it is advantageous to use the 20-scale. Each main division on that scale then represents 1 chain, each subdivision represents a tenth of a chain, and hundredths of a chain can be estimated closely. The starting point of the traverse, or corner 1, is located at an actual distance of $3\frac{1}{4}$ inches from the lower border line.

50. The first step in plotting the traverse in Fig. 5 is to lay off on the meridian from corner 1 a distance 1-*A* equal to the latitude of course 1-2, or 4.24 chains. This distance is laid off upward because the course 1-2 bears in a northerly direction and its latitude is positive. At the point *A* on the meridian a perpendicular to that meridian is drawn toward the right because the course 1-2 bears in an easterly direction and its departure is positive. Corner 2 is then located on the perpendicular at a distance from *A* equal to the departure of course

1-2 or 4.24 chains. As a check, the plotted length of the course 1-2 should be measured. If it is not exactly equal to the value of 6 chains recorded in Table VI, one or more errors were made in the plotting, and the necessary corrections should be made before the next course is plotted. When the correct position of corner 2 has been established, course 1-2 is drawn on the plate.

To locate corner 3, the procedure is as follows: A line representing the meridian is drawn downward from corner 2, because the course 2-3 runs in a southerly direction and its latitude is negative. Along the meridian, the distance 2-B is made equal to the latitude of course 2-3, or 2.18 chains. Then a perpen-

TABLE VII
NOTES FOR TRAVERSE IN FIG. 6 OF PLATE 801

Station	Deflection Angle	Bearing
25 + 80	End of line	
20 + 38	37° 20' L.	
15 + 18	31° 08' L.	
9 + 13	39° 26' R.	
3 + 60	30° 30' R.	
0 + 00		N 45° 00' E

dicular to the meridian is drawn to the right or east from *B* and a distance *B-3* of 4.78 chains, which is the departure of course 2-3, is laid off on the perpendicular. The length of the course 2-3 thus determined is checked, and, if it is found to be correct, the plotted positions of corners 2 and 3 are joined by a straight line.

The steps in plotting course 3-4 are similar to those described for course 1-2, but the latitude 3-C and departure C-4 of course 3-4 are, respectively, 4.17 and 1.99 chains. Also, course 4-5 is plotted in the same manner as course 2-3, the latitude 4-D being 2.63 chains and the departure D-5 being 7.31 chains. Since the course 5-6 runs in a southwesterly direction, its latitude of 4.75 chains is laid off downward from corner 5; and, from the point *E* thus established on the meridian, the departure E-6 of 2.83 chains is laid off to the left or west.

51. Plotting Deflection Angles by Method of Chords.—The notes in Table VII for plotting the traverse in Fig. 6 are similar to those in Tables II and IV, but the deflection angles in this case are to be laid off by the method of chords, described in Art. 24. The starting point *A*, or Sta. 0 + 00, is located $\frac{1}{2}$ inch from the lower border line. The direction of the first course from Sta. 0 + 00 to Sta. 3 + 60 is established on the plate in the following manner: Along the meridian line through point *A*, the point *B* is located at a distance of 5 units from *A*; a convenient length for a unit is one main division on the 20-scale. Since the bearing of the first course is N 45° 00' E, the chord from the point *B* to the required course lies to the right of the meridian and its length is $2 \times 5 \times \sin \frac{1}{2} \times 45^\circ = 3.83$ units. Hence, the point *C* on the course is located at the intersection of an arc *DE* that has *A* as its center and 5 units as its radius, and an arc *FG* that has *B* as its center and 3.83 units as its radius. Then, a straight line is drawn through *A* and *C*; and point *H*, or Sta. 3 + 60, is established on that line at a distance of 360 feet from *A*, to a scale of 1 inch = 200 feet.

52. The procedure for plotting the course in Fig. 6 from Sta. 3 + 60 to Sta. 9 + 13 is similar to that just described for the first course. Point *I* is located on the prolongation of course *AH* and at a distance of 5 units from *H*; and the required length of the chord *IJ* is $2 \times 5 \times \sin \frac{1}{2} \times 30^\circ 30' = 2.63$ units. After point *J* has been located to the right of *HI* at the intersection of arcs swung with *H* and *I* as centers, and with radii of 5 and 2.63 units, respectively, the line *HJ* is prolonged and point *K*, or Sta. 9 + 13, is plotted at a distance of $913 - 360 = 553$ feet from *H*.

In plotting the course from Sta. 9 + 13 to Sta. 15 + 18, the first step is to locate point *L* along the extension of the preceding course *HK* and at a distance of 5 units from *K*. The next step is to establish point *M* at the intersection of two arcs, of which one has *K* as a center and 5 units as a radius and the other has *L* as a center and $2 \times 5 \times \sin \frac{1}{2} \times 39^\circ 26' = 3.37$ units as a radius. On the line through *K* and *M*, the point *N*,

or Sta. 15 + 18, is located at a distance of $1,518 - 913 = 605$ feet from K .

A similar procedure is followed for plotting each of the other two courses of the traverse, but the deflection angles are to the left. The length of chord OP should be $2 \times 5 \times \sin \frac{1}{2} \times 31^\circ 08' = 2.68$ units, and the point P should lie to the left of the prolongation NO . Also, the distance from N to point Q , or Sta. 20 + 38, should be $2,038 - 1,518 = 520$ feet.

The required length of chord RS is $2 \times 5 \times \sin \frac{1}{2} \times 37^\circ 20' = 3.20$ units and the distance from Q to T , or Sta. 25 + 80, is $2,580 - 2,038 = 542$ feet.

53. Finishing Plate.—After all six figures on Plate 801 been drawn lightly, the vertexes of the angles along the traverses should be marked by small circles and the courses made fairly heavy. The diameter of the circles should preferably be about $\frac{1}{16}$ inch and should not exceed $\frac{3}{32}$ inch. The courses should not be continued through the circles at their extremities, but should be stopped accurately at the perimeters of the circles. Also, the station numbers (or, in Fig. 5, the lengths of the lines and the corner numbers) and the angles or the bearings should be inserted. Finally, the figure numbers, the meridian arrow, the title at the top of the plate, the border lines, and the information in the lower right-hand corner should be added. As shown on the sample plate, upright modern Roman capitals should be used for lettering the title, PLOTTING OPEN TRAVERSES. Single-stroke vertical lettering should be used for the figure numbers, the notes concerning the scales, and the information in the 1 in. \times 3 in. rectangle in the lower right-hand corner. The lettering for the station numbers, bearings, deflection angles, and lengths may be either single-stroke slant lettering, as on the sample plate, or single-stroke vertical lettering.

The arrow representing the meridian is located $\frac{3}{4}$ inch from the left-hand border line. Its top should be about 2 inches from the upper border line, and its total length should be approximately $2\frac{1}{2}$ inches. It should have the same proportions as the vertical arrow in Fig. 15 (a). As previously explained,

Ref. page 28 - LEAVE LIGHT CONSTRUCTION LINES.
the student should not dot the construction lines, and should not show on his plate the values in parentheses or the letters that represent points on the sample plate.

MAPPING PLATE 802: PLOTTING CLOSED TRAVERSES

54. Methods of Plotting Closed Traverses.—Any of the methods used in Plate 801 for plotting open traverses can also be applied to closed traverses. However, it is generally more convenient and more accurate to plot a closed traverse either by the method of coordinates described in Art. 40 or by a modification of the usual method of tangents and cotangents. On Plate 802 are plotted two closed traverses. The corners of the traverse in Fig. 1 are located by their coordinates with respect to a meridian and a parallel of latitude through one corner. In the case of the traverse in Fig. 2, the directions of the courses are determined by the tangents or cotangents of their azimuths, and the lengths of the courses are laid off from corners that have been previously plotted. A scale of 200 feet to the inch is to be used for both figures. Also, the meridian is assumed to be parallel to the shorter pair of opposite border lines.

As in the case of Plate 801, construction lines are indicated on the sample plate by dotted lines. These lines should be shown as full lines in pencil on the student's plate, whereas the lines shown full on the sample plate are made heavy by the student. Also, the distances enclosed in parentheses and the letters indicating construction points on the sample plate are for reference only and should not be shown on the student's drawing.

55. Preliminary Calculations for Plotting by Coordinates. Before the corners of a traverse can be plotted by coordinates, it is necessary to calculate the required distances from the directions and lengths of the courses that are returned in the field notes. The first step in this work is to compute, by the principles explained in Art. 39, the latitude and departure of each course from its length and the angle that it makes with the meridian.

In order that the courses of a traverse may form a closed figure, the sum of the east departures must equal the sum of the west departures and the sums of the north and south departures must likewise be equal. However, in an actual survey the field measurements are never exactly correct. It is therefore necessary to balance the survey by adjusting the computed departures and latitudes, and possibly also the original lengths and azimuths, so that the requirements of a closed survey will be met.

The coordinate axes may be passed through any corner of the survey. Generally, however, an inspection of the latitudes and departures will make it possible to choose a corner that will be most advantageous from the standpoint of plotting. The best location will depend somewhat on the shape of the traverse. When the positions of the coordinate axes have been established, the coordinates of the various corners of the traverse are determined by combining the latitudes and departures of the courses in the proper manner. The distance from the meridional axis, or the axis that corresponds to a meridian, is commonly designated as the *X coordinate*; and the distance from the latitudinal axis, or the axis that corresponds to a parallel of latitude, is the *Y coordinate*.

The most serious objection to the method of plotting by coordinates is the amount of calculation involved in the preliminary work. However, where the latitudes and departures of the courses must be computed for other purposes, the additional labor required for calculating the coordinates is not great.

56. Procedure in Plotting by Coordinates.—An inspection of the coordinates will enable the draftsman to establish the direction of the meridian on the map; to determine either the proper scale of the map or, if the scale is already established, the required size of the map; and to decide on the general position of the traverse on the sheet. The first step in plotting is to draw the coordinate axes. There are several possible methods for locating a point by the use of its coordinates; but, in view of the fact that it is usually desirable to obtain the highest degree of accuracy when this method of plotting is used, only the most accurate procedure is here described.

A series of equal squares of convenient size should be constructed by drawing lines that are parallel to the coordinate axes. However, if the area covered by the plotted traverse is small, a single square may be sufficient. The accuracy of each square is checked by measuring the lengths of its two diagonals and comparing these distances with the value computed by multiplying the theoretical length of a side of the square by 1.414. Any corner in the traverse is then plotted best by measuring distances from the most convenient sides of the square in which that corner is located. The size of the squares for any particular map depends on the number of corners to be located, on the positions of the corners, and on the scale of the map.

The details of the procedure for plotting a point by coordinates are illustrated in Fig. 21. In this case, it is assumed that corner number 4 of a traverse is 816.3 feet east of the meridional axis and 1,187.7 feet north of the latitudinal axis, and the parallel lines forming the squares are located at intervals of 500 feet. The lines that bound the square within which the corner lies are designated as *E 500*, *E 1,000*, *N 1,000*, and *N 1,500*, which are, respectively, 500 and 1,000 feet east of the meridional axis and 1,000 and 1,500 feet north of the latitudinal axis. To plot the corner, either of two methods may be adopted.

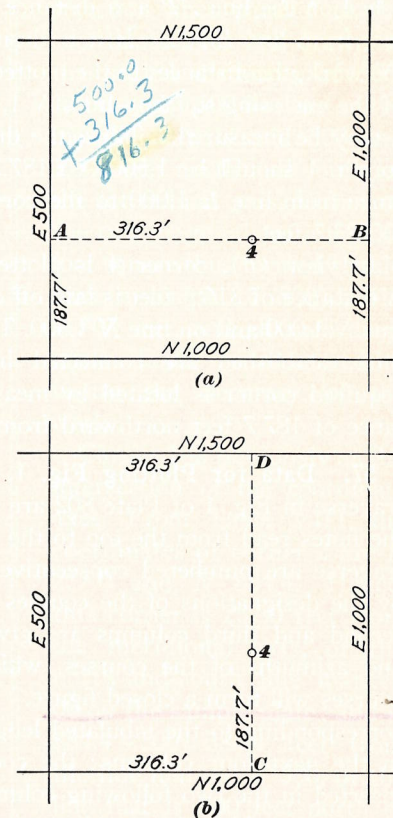


Fig. 21

In view (a), the distance of $1,187.7 - 1,000 = 187.7$ feet is laid off along each meridional boundary of the square northward from line $N\ 1,000$; and the points A and B thus located are joined by a straight line, every point on which has a Y coordinate of $1,187.7$. Finally, the required corner is established on the line AB at a distance of $816.3 - 500 = 316.3$ feet east from line $E\ 500$. If it is desired to check the accuracy of the work, the distances to the plotted corner from the other sides of the enclosing square—lines $N\ 1,500$ and $E\ 1,000$, in this case—may be measured. Thus, the distance from line $N\ 1,500$ to corner 4 should be $1,500 - 1,187.7 = 312.3$ feet, and the distance from line $E\ 1,000$ to the corner should be $1,000 - 816.3 = 183.7$ feet.

In view (b), corner 4 is plotted in the following manner: A distance of 316.3 feet is laid off eastward from line $E\ 500$ on line $N\ 1,000$ and on line $N\ 1,500$. Then the two points C and D thus established are connected by a straight line, and the required corner is located by measuring along that line a distance of 187.7 feet northward from line $N\ 1,000$.

57. Data for Plotting Fig. 1.—The data for plotting the traverse in Fig. 1 of Plate 802 are shown in Table VIII. Here, the notes read from the top to the bottom. The corners of the traverse are numbered consecutively from 1 to 6, as indicated by the designations of the courses in the first column. In the second and third columns are given, respectively, the lengths and azimuths of the courses, which are adjusted so that the courses will form a closed figure. The departures and latitudes corresponding to the tabulated lengths and azimuths are shown in the next four columns; the coordinates of the corners are inserted in the two following columns; and the numbers of the corners to which the coordinates apply are entered in the last column. The coordinate axes pass through corner 1, as indicated by the fact that both coordinates of corner 1 are zero.

58. Instructions for Plotting Fig. 1.—The traverse in Fig. 1 is plotted in the following manner: With the short dimension of the plate vertical, the meridian through corner 1 is located at an actual distance of 1 inch from the left-hand border

TABLE VIII
NOTES FOR TRAVERSE IN FIG. 1 OF PLATE 802

Course	Length Feet	Azimuth	Departure		Latitude		Coordinates		To GET Corner #
			E	W	N	S	X	Y	
1 to 2	915.6	15° 00'	237.0		884.4		E 237.0	N 884.4	2
2 to 3	1,130.3	96° 12'	1,123.7			122.1	E 1,360.7	N 762.3	3
3 to 4	725.9	215° 37'		422.8		590.1	E 937.9	N 172.2	4
4 to 5	900.6	164° 05'	247.0			866.1	E 1,184.9	S 693.9	5
5 to 6	828.0	249° 50'		777.2		285.5	E 407.7	S 979.4	6
6 to 1	1,060.9	337° 24'		407.7	979.4		0.0	0.0	1

line, and the parallel of latitude through that corner is located midway between the upper and lower border lines, or $6\frac{1}{2}$ inches from either line. Also, construction lines parallel to the coordinate axes are drawn at the scaled distances of 500, 1,000, and 1,500 feet east of the meridional axis and 500 and 1,000 feet north and south of the latitudinal axis. Either of the two diagonals of any 500-foot square thus formed should have a length of $500 \times 1.414 = 707$ feet, and all such diagonals should be measured to insure the accuracy of the work.

Corner 1 is obviously at the intersection of the coordinate axes. From Table VIII, the coordinates of corner 2 are $E\ 237.0$ and $N\ 884.4$. Hence, this corner is 237.0 feet from the meridional axis, or the north-and-south line marked 00 on the sample plate; and is $884.4 - 500 = 384.4$ feet from the line marked $N\ 500$, or the line that is 500 feet north of the latitudinal axis. Corner 2 is located on the plate in the following manner: First, point A is plotted by laying off along the meridional axis a distance of 384.4 feet northward from line $N\ 500$, and point B is

plotted on line *E* 500 at the same distance from line *N* 500. Then, a straight line is drawn between points *A* and *B*; and corner 2 is established on that line at a distance of 237.0 feet from *A*. The length of the course 1-2 should be scaled and compared with the value 915.6 given in the second column of Table VIII. If the plotted position of corner 2 is found to be correct, that point should be connected with corner 1 by a straight line.

Since the coordinates of corner 3 are *E* 1,360.7 and *N* 762.3, that corner is $1,360.7 - 1,000 = 360.7$ feet east of line *E* 1,000 and $762.3 - 500 = 262.3$ feet north of line *N* 500. Hence, it is located by measuring 262.3 feet northward from line *N* 500 along line *E* 1,000 to point *C* and also along line *E* 1,500 to point *D*; drawing a straight line between *C* and *D*; and laying off along that line a distance of 360.7 feet from *C*. This position of corner 3 is then checked by scaling the length of course 2-3 and comparing that length with the value 1,130.3 recorded in Table VIII.

Corners 4, 5, and 6 are located by following a similar procedure. To determine the position of corner 4, the *Y* coordinate of 172.2 feet is first measured, northward from the latitudinal axis, along line *E* 500 to point *E* and along line *E* 1,000 to point *F*. Then, a straight line is drawn between points *E* and *F*, and a distance of $937.9 - 500 = 437.9$ feet is laid off along that line from *E*. The location of corner 5 is found by first establishing point *G* on line *E* 1,000 and point *H* on line *E* 1,500 at a distance of $693.9 - 500 = 193.9$ feet south of line *S* 500; and then connecting *G* and *H* and measuring 1,184.9 - 1,000 = 184.9 feet to the east from *G*. For the last corner, 6, points *I* and *J* are plotted on the meridional axis and on line *E* 500 by laying off a distance of $979.4 - 500 = 479.4$ feet southward from line *S* 500. Then, the *X* coordinate of 407.7 feet is measured along the line *IJ* from point *I*. The lengths of the courses 3-4, 4-5, 5-6, and 6-1 are checked and these courses are plotted.

USING COORDINATES.
Since an error in the position of one corner does not affect the positions of the other corners, it is permissible to plot all the corners before the lengths of the courses are checked.

However, if desired, each corner can be checked as soon as it is plotted by comparing the scaled distance from the preceding corner with the given length of the course.

59. Plotting Azimuths or Bearings by Modified Method of Tangents and Cotangents.—When a traverse occupies a comparatively small area on a map and it is not desired to compute

TABLE IX
SIDES OF SQUARE USED FOR PLOTTING AZIMUTHS OR BEARINGS

Azimuth of Course From North	Bearing of Course	Side of Square Used as Long Leg of Triangle	Multiplier of 10 Function of Angle Between Meridian and Course	Side of Square Along Which Short Leg of Triangle is Measured	Corner of Square From Which Hypotenuse of Triangle Starts
0° to 45°	Due North to N 45° E	West	Tangent	North	Southwest
45° to 90°	N 45° E to Due East	South	Cotangent	East	Southwest
90° to 135°	Due East to S 45° E	North	Cotangent	East	Northwest
135° to 180°	S 45° E to Due South	West	Tangent	South	Northwest
180° to 225°	Due South to S 45° W	East	Tangent	South	Northeast
225° to 270°	S 45° W to Due West	North	Cotangent	West	Northeast
270° to 315°	Due West to N 45° W	South	Cotangent	West	Southeast
315° to 360°	N 45° W to Due North	East	Tangent	North	Southeast

the coordinates of the corners, the directions of the courses can be determined conveniently by the following method: A large square, each side of which has a length of 10 units, is laid out so that its center is near the center of the sheet or near the center of the portion of the sheet on which the traverse is to be plotted. Then the angle that each course makes with the meridian is laid off within this square by the tangent or cotangent of the angle, as described in Art. 21, 22, or 23.

sides of the square. However, it is important that the extremities of the hypotenuse of each right triangle be definitely identified, so that there will be no doubt or confusion as to the directions of the courses when they are being plotted on the map. Numbers corresponding to those of the corners of the traverse are therefore inserted temporarily at the corners of the square and at the points along its sides.

TABLE X
NOTES FOR TRAVERSE IN FIG. 2 OF PLATE 802

Course	Length Feet	Azimuth	Legs of Triangle	
			Parallel to Meridian	Perpendicular to Meridian
1 to 2	563.3	36° 23'	10 W ✓	7.37 N →
2 to 3	877.0	100° 01'	1.77 E ↓	10 N ✓
3 to 4	680.0	335° 40'	10 E	4.52 N ←
4 to 5	541.8	52° 46'	7.60 E ↑	10 S ✓
5 to 6	1,005.2	313° 04'	9.35 W ↑	10 S ✓
6 to 7	550.2	245° 29'	4.56 W ↓	10 N ✓
7 to 8	485.6	207° 30'	10 E ✓	5.21 S ←
8 to 9	527.5	151° 51'	10 W ✓	3.28 S →
9 to 1	776.3	184° 00'	10 E	0.70 S

The number of each corner of the traverse appears twice, once at a corner of the square and once on a side of the square. When the legs of the triangle for a course are about to be established, the corner of the square from which the hypotenuse of the triangle is to start is given the same number as the traverse corner at the beginning of the course. As soon as the short leg of the triangle is laid off, the point at the end of that leg, which point is also at the end of the hypotenuse, is marked with the number of the traverse corner that is at the end of the course. For example, if the course whose direction is indicated by the line *AE*, Fig. 22, extends from corner 1 to corner 2 of a traverse, the number 1 would be placed at the southwest corner *A* of the square and the number 2 would be assigned to the point *E* on the north side of the square, as shown. Similarly, if the direction of the course 2-3 of the traverse were indicated by the line *DF*, the number 2 would be placed at the southeast

corner *D* of the square, and the point *F* on the west side would be marked 3. If it is desired to provide an additional safeguard against later doubt, the hypotenuse of the triangle may also be drawn, as in Fig. 22. But the traverse can be plotted without drawing any of the hypotenuses in the square.

62. Notes for Traverse in Fig. 2.—The traverse in Fig. 2 of Plate 802 is plotted from the notes in Table X by the method of Art. 59. In the first column of the table are given the courses and in the second and third columns are given their lengths and azimuths, as adjusted to give a closed traverse. In the last two columns are tabulated the lengths and positions of the legs of the right triangles for the various courses. The numerical value of each leg shows the number of units to be laid off and the letter indicates the side of the square along which the measurement is to be made. Thus, 4.52 *N* means 4.52 units on the north side of the square. In Fig. 2, the length of a unit is taken as one main division on the 20-scale.

63. Construction of Square in Fig. 2.—The square that is used for determining the directions of the courses in Fig. 2 is located so that its right-hand side is $1\frac{3}{4}$ inches from the right-hand border line and its upper edge is 4 inches from the top border line. Each side of the square has a length of 10 units, or 10 main divisions on the 20-scale.

As given in Table X, the azimuth of course 1-2 is 36° 23'. Since this azimuth is between 0° and 45°, it is found from Table IX that the long leg of the triangle is the west side of the square; that the short leg is equal to 10 times the tangent of the angle, and is laid off along the north side of the square; and that the hypotenuse starts from the southwest corner of the square. Also, the length of the short leg of the triangle is $10 \times \tan 36^\circ 23' = 10 \times 0.737 = 7.37$ units, as shown in Table X. Therefore, the first step in establishing the direction of course 1-2 in the square is to assign to the southwest corner of the square the number 1. Then, the distance of 7.37 units is measured along the north side of the square from the northwest corner, and the end of that measurement is given the number 2.

If desired, the hypotenuse from corner 1 to point 2 may be drawn, as on the sample plate; or it may be omitted.

Since the azimuth of course 2-3 is $100^{\circ} 01'$, that course makes an angle of $180^{\circ} - 100^{\circ} 01' = 79^{\circ} 59'$ with the meridian. Also, since the azimuth is between 90° and 135° , the long leg of the triangle is the north side of the square; and the northwest corner is marked 2. The short leg is measured along the east side from the northeast corner and has a length of $10 \times \cot 79^{\circ} 59' = 10 \times 0.177 = 1.77$ units; and the end of that leg is marked 3.

The next course to be considered is 3-4. Since its azimuth $335^{\circ} 40'$ is between 315° and 360° , the long leg of the triangle is the east side of the square, and the southeast corner is marked 3. Also, the short leg is equal to $10 \times \tan (360^{\circ} - 335^{\circ} 40') = 10 \times 0.452 = 4.52$ units, and is laid off along the north side of the square from the northeast corner. The point thus located on the north side is marked 4.

For course 4-5, the long leg of the triangle is the south side of the square, and the southwest corner is also marked 4. The short leg, which is measured along the east side, is equal to $10 \times \cot 52^{\circ} 46' = 10 \times 0.760 = 7.60$ units; and the end of the measurement is marked 5.

64. The directions of the other courses are established by applying the data in Table IX in a similar manner. For course 5-6, the long leg of the triangle is the south side of the square; and the short leg has a length of $10 \times \cot (360^{\circ} - 313^{\circ} 04') = 10 \times 0.935 = 9.35$ units, and is measured along the west side. The triangle for course 6-7 has as its long leg the north side of the square. Its short leg is in the west side and has a length of $10 \times \cot (245^{\circ} 29' - 180^{\circ}) = 10 \times 0.456 = 4.56$ units.

In the triangle for course 7-8, the long leg is the east side of the square; and the short leg, which is equal to $10 \times \tan (207^{\circ} 30' - 180^{\circ}) = 10 \times 0.521 = 5.21$ units, is measured along the south side. For course 8-9, the long leg of the triangle is the west side of the square and the short leg, in the south side, is equal to $10 \times \tan (180^{\circ} - 161^{\circ} 51') = 10 \times 0.328 = 3.28$ units. Finally, in the triangle for course 9-1, the long leg is

the east side of the square and the length of the short leg, which is in the south side, is $10 \times \tan (184^{\circ} 00' - 180^{\circ}) = 10 \times 0.070 = 0.70$ unit.

65. Procedure in Plotting Traverse in Fig. 2.—The direction of each course of the traverse plotted in Fig. 2 of Plate 802 is determined by transferring to the proper position in the plot the direction established in the construction square. The instruments used for this purpose may be a pair of triangles. Corner 1 of the traverse is located $1\frac{1}{2}$ inches from the lower border line of the plate and $7\frac{1}{2}$ inches from the right-hand border line. To locate corner 2, the first step is to draw from corner 1 a line having the direction of course 1-2, as determined by the positions of the proper points marked 1 and 2 in the square. Then, the required length of course 1-2, or 563.3 feet, is laid off along this line from corner 1. In a similar manner, corner 3 is located from corner 2 by drawing from the latter corner a line having the direction of course 2-3 and laying off the length of the course, or 877.0 feet, from corner 2.

The other corners are established by plotting the various courses in numerical sequence. In each case, the course is made parallel to the corresponding line within the square, and the length is taken from the notes in Table X. As a check on the work, the course from corner 9 to corner 1 should be plotted in the same manner as the other courses and the position of corner 1 thus determined should coincide with the point from which course 1-2 was started.

66. Finishing Plate 802.—On the student's plate, the courses should be heavy; and the construction lines, which are shown dotted on the sample plate, should be made lightly. Also, the numbers of the corners on the traverses and the lengths and the directions of the courses should be inserted but, the distances enclosed in parentheses and the reference letters shown on the sample plate should not be put on the drawing. A true-meridian arrow should be drawn in the upper right-hand corner of the sheet, its size and position being about in the same proportion as on the sample plate. The title, the figure numbers, the border lines, and the information in the lower right-hand

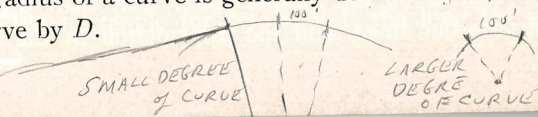
corner of the plate complete the drawing. The title should be in upright modern Roman capitals; the figure numbers, the note regarding the scale, and the information in the rectangle in the lower right-hand corner should be in single-stroke vertical letters; and the lengths and azimuths of the courses may be in either single-stroke slant letters or single-stroke vertical letters.

ROUTES INVOLVING CIRCULAR CURVES

ELEMENTS OF CIRCULAR CURVES

67. **Railroad or Highway Location.**—Because of the irregularity of the ground surface and the presence of natural and artificial obstacles, the route for a railroad or highway consists of a succession of straight lines that are connected by curves. The straight lines are tangent to the curves and are therefore known as *tangents*. The curve joining two tangents may be of uniform radius for its entire length, and is then called a *simple curve*; or it may be composed of two or more parts with different radii. A continuous curve that is composed of two or more circular arcs curving in the same direction but with different radii is known as a *compound curve*. Often a gradual transition between a circular curve and a tangent is effected by means of a *spiral*, or *easement*, curve, whose radius changes gradually.

BAKER 68. **Radius and Degree of Curve.**—The form of a circular curve, or the sharpness with which it turns, may be designated by its radius. However, in railroad or highway work, the usual method of indicating the sharpness of such a curve is by giving the *degree of curve*. Most railroad engineers and some highway engineers use as the degree of curve the central angle subtended by a chord 100 feet long, that is, the angle formed at the center of the curve by two radii passing through the extremities of a 100-foot chord. Many highway engineers use as the degree of curve the central angle subtended by an arc that is 100 feet in length. The radius of a curve is generally denoted by R , and the degree of curve by D .



In Fig. 23, the tangents AB and CE are connected by the simple curve FG , and it is assumed that the successive points H , J , K , and L on the curve are 100 feet apart. Thus, the angle D that is formed at the center O by the radii to any two consecutive 100-foot stations is the degree of curve. If the angle D is 4° , the curve is called a 4° curve; and if the angle is $1^\circ 30'$, the curve is a $1^\circ 30'$ curve. Whether the distance of 100 feet would be measured along a chord or along an arc would depend on the basis for the degree of curve.

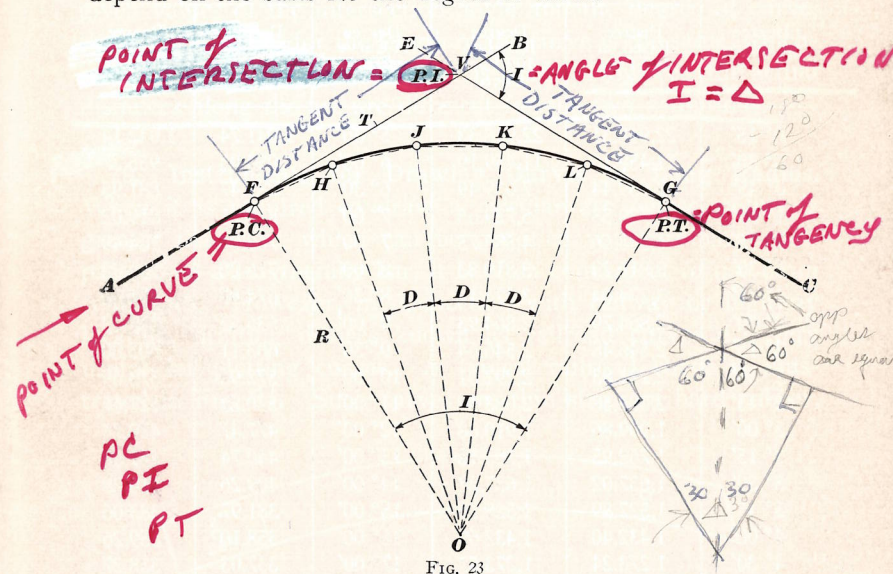


FIG. 23

In Table XI are given the radii corresponding to various degrees of curve. The values in the second column are for use where the degree of curve is based on a 100-foot arc; and those in the third column are for use where the degree is based on a 100-foot chord.

DAVIS 69. **Intersection Angle, Tangent Distance, and Length of Curve.**—The point at which the two tangents through the extremities of a curve intersect is known as the *vertex*, *point of intersection*, or *P.I.* The external angle formed by the intersecting tangents, as angle BVG in Fig. 23, is called the *inter-*

section angle, or angle of intersection, and is commonly denoted either by I or by the Greek letter Δ (delta). For any curve, the central angle between the radii to the extremities of the curve is equal to the intersection angle. Thus, the angle FOG is equal to I .

TABLE XI
RADII OF CURVES

Degree of Curve	Radius, in Feet		Degree of Curve	Radius, in Feet	
	Degree Based on 100-Foot Arc	Degree Based on 100-Foot Chord		Degree Based on 100-Foot Arc	Degree Based on 100-Foot Chord
0° 15'	22,918.31	22,918.33	5° 30'	1,041.74	1,042.14
0° 30'	11,459.16	11,459.19	6° 00'	954.93	955.37
0° 45'	7,639.44	7,639.49	6° 30'	881.47	881.95
1° 00'	5,729.58	5,729.65	7° 00'	818.51	819.02
1° 15'	4,583.66	4,583.75	7° 30'	763.94	764.49
1° 30'	3,819.72	3,819.83	8° 00'	716.20	716.78
1° 45'	3,274.04	3,274.17	8° 30'	674.07	674.69
2° 00'	2,864.79	2,864.93	9° 00'	636.62	637.27
2° 15'	2,546.48	2,546.64	9° 30'	603.11	603.80
2° 30'	2,291.83	2,292.01	10° 00'	572.96	573.69
2° 45'	2,083.48	2,083.68	11° 00'	520.87	521.67
3° 00'	1,909.86	1,910.08	12° 00'	477.47	478.34
3° 15'	1,762.95	1,763.18	13° 00'	440.74	441.68
3° 30'	1,637.02	1,637.28	14° 00'	409.26	410.28
3° 45'	1,527.89	1,528.16	15° 00'	381.97	383.06
4° 00'	1,432.40	1,432.69	16° 00'	358.10	359.26
4° 30'	1,273.24	1,273.57	17° 00'	337.03	338.27
5° 00'	1,145.92	1,146.28	18° 00'	318.31	319.62

The point where the curve begins, or where the first tangent meets the curve, is called the *point of curve*, or simply the *P.C.*; and the point where the curve ends, or meets the second tangent, is the *point of tangency*, or the *P.T.* The *P.C.* and the *P.T.* are equidistant from the *P.I.*, and the straight line distance from the *P.C.* or the *P.T.* to the *P.I.* is known as the *tangent distance*. This distance is usually denoted by T . In the case of a compound curve, the point at which the curvature changes is called the *point of compound curve*, or the *P.C.C.*

Where the degree of a simple curve is based on an arc of 100 feet, the length of the curve is the distance from the *P.C.* to the *P.T.* measured along the curve. If the degree of curve is based on a 100-foot chord, the length of the curve is measured along a series of successive chords joining the 100-foot stations on the curve. Thus, if the basis of the degree of the curve in Fig. 23 is a 100-foot chord, the length of the curve is the sum of the lengths of the chords FH , HJ , JK , KL , and LG . For given values of the intersection angle and the degree of curve, the working value for the length of the curve is the same, whichever definition of degree of curve is used. However, the actual distances along the arc are different for the two definitions.

70. Instruments for Drawing Curves of Long Radius.

When it is required to draw a circular arc whose radius is greater than that which can be obtained with ordinary compasses and a lengthening bar, it is often convenient to use beam compasses. However, in connection with railway or highway location, drafting devices known as railroad curves are very useful for plotting curves. A typical railroad curve is illustrated in Fig. 24. Such curves are thin strips of hard rubber,

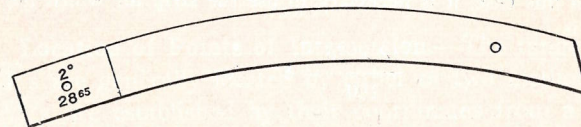


FIG. 24

pear-wood, celluloid, or metal. They usually come in sets, each set containing a number of strips whose edges are curved to different degrees of curve; but, all the radii in a particular set are to the same scale, which is usually 100 feet to the inch. The inner and outer edges of each strip have the same radius. The degree of curve of the edges and the true length of the radius in inches are generally stamped distinctly on the strip, as shown in the illustration. For plotting purposes, the same curve can be used whether the degree of curve is the angle subtended by a chord of 100 feet or the angle subtended by an arc of 100 feet.

In view of the limit of accuracy obtainable in plotting, the degree of curve may be assumed to be inversely proportional to the radius. Since the actual radius of a curve on a map is proportional to the number of feet to the inch in the scale of the map, a single set of railroad curves can be readily adapted to several different scales. If it is required to draw a curve of a certain degree to a certain scale, the required degree of curve to the scale of the available set of curves may be found by the following rule: $(\text{GIVEN } \circ \times \text{ft/in. map scale}) \div (\text{ft/in. R.R. CURVE})$

Rule.—Multiply the specified degree of curve by the number of feet to the inch in the scale of the map and divide the product by the number of feet to the inch in the scale of the set of curves.

EXAMPLE.—A set of railroad curves is cut to the scale of 100 feet to the inch. What curve of that set should be used (a) for a 5° curve on a map for which the scale is 300 feet to the inch, and (b) for a 12° curve on a map whose scale is 50 feet to the inch?

SOLUTION.—(a) The specified degree of curve is 5°, the scale of the map is 300 ft. to the inch, and the scale of the available set of curves is 100 ft. to the inch. Hence, the degree of the strip to be used in drawing the arc on the map is

$$\frac{5 \times 300}{100} = 15^\circ. \text{ Ans.}$$

(b) In this case, it is necessary to use the strip for which the degree of curve is

$$\frac{12 \times 50}{100} = 6^\circ. \text{ Ans.}$$

71. General Procedure in Plotting Route With Curves

When a route consisting of tangents and curves is to be plotted, it is usually advisable to locate the tangents and their intersection points first, just as if there were no curves. The P.C. and P.T. of each curve are then established by measuring the tangent distance for that curve from the P.I. along the two tangents to the curve. Finally, the curves are drawn either by means of railroad curves or by locating the center of the curve and using compasses. Thus, in the case of the route shown in part in Fig. 25, the tangents AB, BC, and CD would be located first; the extremities E, F, G, and H of the curves would be established next by measuring the proper tangent distances from

the P.I.'s at B and C; and the curves EF and GH would then be drawn.

When the scaled length of the radius of a curve is short enough to permit the use of compasses for drawing in the curve, the center of the curve may be readily located at the intersection of two arcs described with the P.C. and the P.T. of the curve as centers and the radius of the curve as a radius. For example, in Fig. 25, the center I of the curve EF may be located by describing the arcs JK and LM with the points E and F as centers and the radius of the curve as a radius. Then the curve is drawn with the same radius and point I as a center.

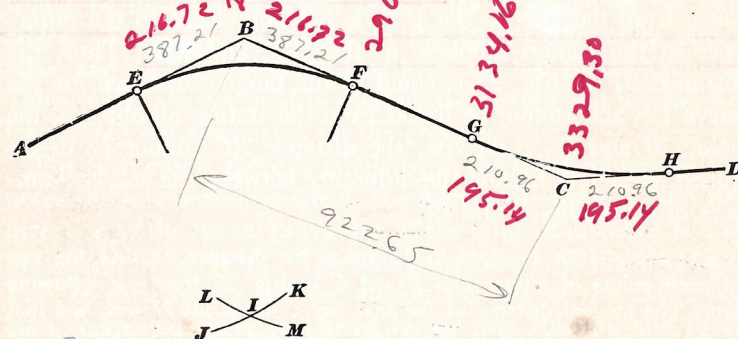


FIG. 25

72. Location of Points of Intersection.—The tangents and the P.I.'s are generally located by either of two methods: (1) The P.I.'s are established by their coordinates from a pair of reference axes, and these points are connected by straight lines. (2) The tangents are plotted successively by their lengths and bearings or azimuths.

In either method, it is necessary to know the lengths of the straight lines between the points of intersection of the curves. The distance between the P.I.'s of two adjacent curves is the sum of the tangent distance between the P.I. and the P.T. of the first curve, the distance between the P.T. of that curve and the P.C. of the second curve, and the tangent distance between the P.C. and the P.I. of the second curve. Thus, in Fig. 25, the distance between the P.I. at B and the P.I. at C is equal to BF + FG + GC.

For the purpose of identifying the various P.I.'s along a route, it is convenient to assign a station number to each P.I. by continuing the stationing along the tangent through the P.C., as if no curve were introduced. Hence, the difference between the station numbers of the P.C. and the P.I. of any curve is equal to the tangent distance for the curve. However, the station number of the P.T. of any curve is determined from the station number of the P.C. and the length of the curve. Since the distance from the P.C. to the P.T. along the curve is obviously less than the sum of the distances from the P.C. to the P.I. and from the P.I. to the P.T., it must be kept in mind that the distance from the P.I. to the P.T. is not equivalent to the difference in the station numbers at the P.T. and P.I. Thus, the distance BF in Fig. 25 is equal to the tangent distance for the curve EF and cannot be found from the station numbers of the points B and F . Likewise, the distance BC cannot be determined by merely finding the difference between the station numbers of the points C and B .

EXAMPLE.—It is required to determine the distance between the two adjacent points of intersection B and C in Fig. 25 from the following data: The tangent distance of the first curve EF is 216.72 feet, and that of the second curve GH is 195.14 feet. Also, the station number of the point B is $27 + 18.47$, that of F is $29 + 07.50$, that of G is $31 + 34.16$, and that of C is $33 + 29.30$.

SOLUTION.—The required distance is the sum of BF , FG , and GC . The distance BF is equal to the tangent distance for the curve EF , or 216.72 ft., and is not equal to the difference between the station numbers of the P.I. at B and the P.T. at F . To find the distance FG , however, it is necessary to take the difference between the station numbers of G and F . This distance is $3,134.16 - 2,907.50 = 226.66$ ft. The other distance GC is equal to the tangent distance for the curve GH , or 195.14 ft., but it may also be found from the station numbers of the P.C. at G and the P.I. at C , as follows: $3,329.30 - 3,134.16 = 195.14$ ft. Hence, the total distance from B to C is

$$216.72 + 226.66 + 195.14 = 638.52 \text{ ft. Ans.}$$

The work may be conveniently tabulated as follows:

- ✓ Sta. $27 + 18.47$ to Sta. $29 + 07.50 = 216.72$ ft. (tangent distance)
- ✓ Sta. $29 + 07.50$ to Sta. $31 + 34.16 = 226.66$ (station difference)
- ✓ Sta. $31 + 34.16$ to Sta. $33 + 29.30 = 195.14$ (tangent distance)
- ✓ Sta. $27 + 18.47$ to Sta. $33 + 29.30 = 638.52$ ft. Ans.

MAPPING PLATE 803: PLOTTING ROUTE CENTER LINES

73. Field Notes for Routes Plotted on Plate.—On Plate 803 are plotted two lines representing the center lines of routes that consist of tangents and curves. The notes for the route in Fig. 1 are given in Table XII, and those for the route in Fig. 2 are given in Table XIII. The forms of notes are somewhat different in the two cases, because Table XII applies to a railroad for which the degree of curve is based on a 100-foot chord, whereas Table XIII is for a highway for which the basis for degree of curve is a 100-foot arc. All the information that would be included in actual field notes is shown in the two tables. The deflection angles in the fourth column of either table, and the arc and chord lengths in the last two columns of Table XIII, are not used in plotting the curves, but are needed for staking out the curves on the ground.

For each curve, there is tabulated in the third column of the notes the intersection angle I , the degree of curve D , and the direction of the curve, the radius R , the tangent distance T , and the length of curve L . The station numbers of the P.C. and P.T. of each curve, and the station number of the P.C.C. of each compound curve are listed in the first column, and these points are identified by the entries in the second column. Also, in the second column is entered the station number of the vertex $P.I. = V$ of each curve. The points listed in the second column are not always shown on a completed map.

In the last two columns of Table XII or XIII are given the magnetic bearings of the tangents. The bearings in the first of these two columns are obtained by reading the compass on the transit, and serve chiefly as a check on the intersection angles. Each bearing in the last column is computed from the bearing of the preceding tangent and the intersection angle between the two tangents. The computed bearing of each tangent is the one to be shown on the plot.

74. General Features of Plate.—For both routes shown on Plate 803, the magnetic meridian is assumed to be parallel to the left- and right-hand border lines, and the scale is 300 feet

TABLE XII

NOTES FOR TRAVERSE IN FIG. 1 OF PLATE 803

Station	Point	Description of Curve	Deflection Angle	Magnetic Bearing	
				Observed	Calculated
32 31 + 36.87	P.T.	$I=48^{\circ} 14'$ $D=8^{\circ} L$ $R=716.78'$ $T=320.88'$ $L=602.92'$	24° 07.0' 22° 38.5' 18° 38.5' 14° 38.5' 10° 38.5' 6° 38.5' 2° 38.5'	S 83° 00' E	S 82° 53' E
24 23 + 14.23	P.T.	$I=44^{\circ} 29'$ $D=9^{\circ} R$ $R=637.27'$ $T=260.61'$ $L=494.26'$	22° 14.5' 21° 36.1' 17° 06.1' 12° 36.1' 8° 06.1' 3° 36.1'	S 34° 45' E	S 34° 39' E
22 21 20 19 18 + 19.97	P.C.	1719.97 1342.72 377.25 260.61 637.76	22° 14.5' 21° 36.1' 17° 06.1' 12° 36.1' 8° 06.1' 3° 36.1'	S 79° 00' E	S 79° 08' E
17 16 15 14 13 + 42.72	P.T.	$I=32^{\circ} 38'$ $D=6^{\circ} L$ $R=955.37'$ $T=279.67'$ $L=543.89'$	16° 19.0' 15° 02.1' 12° 02.1' 9° 02.1' 6° 02.1' 3° 02.1'	S 46° 30' E	S 46° 30' E
12 11 10 9 8 + 98.83	P.C.	1078.56 798.83 279.67			

TABLE XII—(Continued)

Station	Point	Description of Curve	Deflection Angle	Magnetic Bearing	
				Observed	Calculated
				<i>for Reference only</i>	<i>use this one</i>
+ 50					
+ 19.74	P.T.	$I=34^{\circ} 03'$ $D=5^{\circ} 30' L$ $R=1,042.14'$ $T=319.11'$ $L=619.09'$	$17^{\circ} 01.5'$ $16^{\circ} 28.9'$ $13^{\circ} 43.9'$ $10^{\circ} 58.9'$ $8^{\circ} 13.9'$ $5^{\circ} 28.9'$ $2^{\circ} 43.9'$	N $11^{\circ} 00'$ E	N $11^{\circ} 04'$ E
+ 00.65	P.C.C.	$I=52^{\circ} 00'$ $D=7^{\circ} L$ $R=819.02'$ $T=399.46'$ $L=742.86'$	$26^{\circ} 00.0'$ $25^{\circ} 58.6'$ $22^{\circ} 28.6'$ $18^{\circ} 58.6'$ $15^{\circ} 28.6'$ $11^{\circ} 58.6'$ $8^{\circ} 28.6'$ $4^{\circ} 58.6'$ $1^{\circ} 28.6'$		
+ 57.79	P.C.				

TABLE XIII

NOTES FOR TRAVERSE IN FIG. 2 OF PLATE 803

Station	Point	Description of Curve	Deflection Angle	Chord	Arc
			NOT USED	FIELD BEARINGS DO NOT USE	COMPUTED BEARINGS USE THESE
35			17° 54.0'	S 36° 30' E	S 36° 39' E
+ 54.85	P.T.		15° 58.8'	54.84	54.85
34		I=35° 48'	12° 28.8'	99.94	100.00
33		D=7° R	8° 58.8'	99.94	100.00
32	PI=32+07.79 V	R=818.51'	5° 28.8'	99.94	100.00
31		T=264.37'	1° 58.8'	99.94	100.00
30		L=511.43'		56.57	56.58
+ 43.42	P.C.				
29				S 72° 30' E	S 72° 27' E
28					
27					
26					
25					
24					
23			22° 10.5'		
+ 05.19	P.T.		22° 01.2'	5.19	5.19
22			19° 01.2'	99.95	100.00
21			16° 01.2'	99.95	100.00
20		I=44° 21'	13° 01.2'	99.95	100.00
19	PI=18+55.23 V	D=6° R	10° 01.2'	99.95	100.00
18		R=954.93'	7° 01.2'	99.95	100.00
17		T=389.21'	4° 01.2'	99.95	100.00
16		L=739.17'	1° 01.2'	99.95	100.00
15				33.98	33.98
+ 66.02	P.C.				
14				S 63° 15' E	N 63° 12' E
13					
12					
11					
10			31° 36'	N	N
+ 40.00	P.T.		30° 00'	39.99	40.00
9			26° 00'	99.92	100.00
8			22° 00'	99.92	100.00
7		I=63° 12'	18° 00'	99.92	100.00
6	PI=5+90.61 V	D=8° R	14° 00'	99.92	100.00
5		R=716.20'	10° 00'	99.92	100.00
4		T=440.61'	6° 00'	99.92	100.00
3		L=790.00'	2° 00'	99.92	100.00
2				49.99	50.00
+ 50.00	P.C.				
1				N	N
0					

START

A

TABLE XIII—(Continued)

Station	Point	Description of Curve	Deflection Angle	Chord	Arc
+ 50					
70					
69					
68					
67					
66					
65					
64					
+ 15.15	P.T.	I=31° 18'	15° 39.0'	15.15	15.15
63		D=9° R	14° 58.1'	99.90	100.00
62	PI=61+45.72 V	R=636.62'	10° 28.1'	99.90	100.00
61		T=178.35'	5° 58.1'	99.90	100.00
60		L=347.78'	1° 28.1'	32.63	32.63
+ 67.37	P.C.				
59					
58					
57					
56					
55					
54					
53					
52	POINT OF TANGENCY				
+ 51.38	P.T.		31° 31.5'	51.37	51.38
51			29° 43.6'	99.94	100.00
50			26° 13.6'	99.94	100.00
49		I=63° 03'	22° 43.6'	99.94	100.00
48		D=7° L	19° 13.6'	99.94	100.00
47	PI=47+52.75 V	R=818.51'	15° 43.6'	99.94	100.00
46		T=502.08'	12° 13.6'	99.94	100.00
45		L=900.71'	8° 43.6'	99.94	100.00
44			5° 13.6'	99.94	100.00
43	POINT OF COMPOUND		1° 43.6'	49.32	49.33
+ 50.67	P.C.C. CURVE		36° 00.0'	50.65	50.67
42			32° 57.6'	49.98	50.00
+ 50			29° 57.6'	49.98	50.00
41			26° 57.6'	49.98	50.00
+ 50			23° 57.6'	49.98	50.00
40			20° 57.6'	49.98	50.00
+ 50	PI=39+97.57 V		17° 57.6'	49.98	50.00
39		I=72° 00'	14° 57.6'	49.98	50.00
+ 50		D=12° L	11° 57.6'	49.98	50.00
38		R=477.47'	8° 57.6'	49.98	50.00
+ 50		T=346.90'	5° 57.6'	49.98	50.00
37	POINT OF CURVE	L=600.00'	2° 57.6'	49.31	49.33
+ 50.67	P.C.				
36					

NOT USED FOR LAYOUT

use these

TABLE XIV
COORDINATES OF INTERSECTIONS OF TANGENTS IN FIG. 1 OF PLATE 803

Tangent	From	To	Length	Bearing	Departure	Latitude	Coordinates		Intersection Point
							X	Y	
	0 + 00	10 + 78.50	1,078.50	S 46° 30' E	E 782.3	S 742.4	E 782.3	S 742.4	10 + 78.50
	10 + 78.50	19 + 80.58	917.53	S 79° 08' E	E 901.1	S 173.0	E 1,683.4	S 915.4	19 + 80.58
	19 + 80.58	27 + 54.83	801.21	S 34° 39' E	E 455.5	S 659.1	E 2,138.9	S 1,574.5	27 + 54.83
	27 + 54.83	37 + 57.25	1,041.26	S 82° 53' E	E 1,033.2	S 129.0	E 3,172.1	S 1,703.5	37 + 57.25
	37 + 57.25	44 + 19.76	718.57	N 45° 07' E	E 509.1	N 507.1	E 3,681.2	S 1,196.4	44 + 19.76
	44 + 19.76	57 + 50	1,349.37	N 11° 04' E	E 259.0	N 1,324.3	E 3,940.2	N 127.9	57 + 50

to the inch. In Fig. 1, the points of intersection of the tangents are located by coordinates with respect to the meridian and parallel of latitude through Sta. 0+00. In Fig. 2, the directions of the tangents are determined by laying off their bearings by the method of tangents or cotangents, and the lengths of the tangents are measured between the points of intersection.

As in the preceding plates, construction lines are shown dotted on the sample plate. Also, angles and distances that are intended only for reference are enclosed in parentheses on the sample plate.

75. Computations for Plotting Fig. 1.—Before the intersections of the tangents in Fig. 1 can be plotted, it is necessary to calculate the coordinates of those points in the manner described in Art. 55 for the closed traverse in Fig. 1 of Plate 802. The necessary data are shown in Table XIV. The station numbers in the columns headed *Tangent* are those of the various intersection points listed in the second column of Table XII. The lengths entered in the col-

umn headed *Length* of Table XIV are computed, in the manner explained in Art. 72, from the tangent distances of the curves and the lengths of the tangents between curves. These tangent distances are given in column 3 of Table XII; and the station numbers of the ends of the curves, from which the distances between curves can be found, are given in the first column of that table. The total distance from Sta. 0 + 00 to Sta. 10 + 78.50 is 1,078.50 feet, as determined from the station numbers, because the stationing from the P.C. at Sta. 7 + 98.83 to the P.I. at Sta. 10 + 78.50 is continued along the initial tangent as if there were no curve. In computing each of the other lengths in the second column of Table XIV, it is necessary to make allowance for the fact that the distance from the P.I. to the P.T. of each curve is equal to the tangent distance for the curve and is not equivalent to the difference between the station numbers of the points. The computations for determining these lengths may be tabulated as follows:

$$\begin{aligned} \text{Sta. 10 + 78.50 to Sta. 13 + 42.72} &= 279.67 \\ \text{Sta. 13 + 42.72 to Sta. 17 + 19.97} &= 377.25 \\ \text{Sta. 17 + 19.97 to Sta. 19 + 80.58} &= 260.61 \\ \text{Sta. 10 + 78.50 to Sta. 19 + 80.58} &= 917.53 \end{aligned}$$

$$\begin{aligned} \text{Sta. 19 + 80.58 to Sta. 22 + 14.23} &= 260.61 \\ \text{Sta. 22 + 14.23 to Sta. 24 + 33.95} &= 219.72 \\ \text{Sta. 24 + 33.95 to Sta. 27 + 54.83} &= 320.88 \\ \text{Sta. 19 + 80.58 to Sta. 27 + 54.83} &= 801.21 \end{aligned}$$

$$\begin{aligned} \text{Sta. 27 + 54.83 to Sta. 30 + 36.87} &= 320.88 \\ \text{Sta. 30 + 36.87 to Sta. 33 + 57.79} &= 320.92 \\ \text{Sta. 33 + 57.79 to Sta. 37 + 57.25} &= 399.46 \\ \text{Sta. 27 + 54.83 to Sta. 37 + 57.25} &= 1,041.26 \end{aligned}$$

$$\begin{aligned} \text{Sta. 37 + 57.25 to Sta. 41 + 00.65} &= 399.46 \\ \text{Sta. 41 + 00.65 to Sta. 44 + 19.76} &= 319.11 \\ \text{Sta. 37 + 57.25 to Sta. 44 + 19.76} &= 718.57 \end{aligned}$$

$$\text{Sta. } 44 + 19.76 \text{ to Sta. } 47 + 19.74 = 319.11$$

$$\text{Sta. } 47 + 19.74 \text{ to Sta. } 57 + 50 = 1,030.25$$

$$\text{Sta. } 44 + 19.76 \text{ to Sta. } 57 + 50 = 1,349.37$$

76. Plotting Intersection Points.—The first step in plotting Fig. 1 is to locate Sta. $0 + 00$ at a distance of $\frac{3}{4}$ inch from the left-hand border line of the plate and $1\frac{1}{2}$ inches from the upper border line; and to draw the coordinate axes through that point. Then, in order to facilitate the measurement of the coordinates of the points of intersection of the tangents, other construction lines are established accurately at intervals of 1,000 feet, to scale. As a check on this work, the diagonals of the squares formed by the coordinate axes and the lines S 1,000, S 2,000, E 1,000, E 2,000, E 3,000, and E 4,000 are measured; these diagonals should measure 1,414 feet, to scale.

The measurements for locating the various intersection points may be conveniently made as follows: For Sta. $10 + 78.50$, the first step is to lay off 782.3 feet eastward along the latitudinal axis from Sta. $0 + 00$ to point A and the same distance along line S 1,000 from the meridional axis to point B . Then points A and B are connected by a straight line; and a distance of 742.4 feet is laid off along that line southward from point A . For Sta. $19 + 80.58$, the distance $1,683.4 - 1,000 = 683.4$ feet is laid off to the east along the latitudinal axis and also along line S 1,000 from line E 1,000 to points C and D , respectively; these points are connected; and 915.4 feet is measured along CD southward from point C . For Sta. $27 + 54.83$, the distance $2,138.9 - 2,000 = 138.9$ feet is laid off along the lines S 1,000 and S 2,000 eastward from line E 2,000 to points E and F , respectively; and on the line EF the distance $1,574.5 - 1,000 = 574.5$ feet is measured southward from point E . To locate Sta. $37 + 57.25$, the distance $3,172.1 - 3,000 = 172.1$ feet is laid off from line E 3,000 along the lines S 1,000 and S 2,000 to points G and H ; and, along the line GH , a distance of $1,703.5 - 1,000 = 703.5$ feet is measured from point G . Similarly, Sta. $44 + 19.76$ is located on the line IJ , which is $3,681.2 - 3,000 = 681.2$ feet from line E 3,000, and at a distance of $1,196.4 - 1,000 = 196.4$ feet from point I . Sta. $57 + 50$ is located by

first establishing points K and L on lines E 3,000 and E 4,000 and 127.9 feet north of the latitudinal axis; and then measuring along line KL a distance of $3,940.2 - 3,000 = 940.2$ feet from K . The position of each point is checked by scaling the distance from the preceding point.

77. Drawing Curves in Fig. 1.—For the first curve in Fig. 1, the P.C. at Sta. $7 + 98.83$ and the P.T. at Sta. $13 + 42.72$ are established by measuring the tangent distance of 279.67 feet along each tangent from the vertex or P.I. at Sta. $10 + 78.50$. Likewise, the P.C. and P.T. of the next curve are located by measuring the tangent distance of 260.61 feet for that curve from its P.I. at Sta. $19 + 80.58$. The ends of the other curves are plotted in a similar manner.

After the ends of all the curves have been located on the plot, each curve is inserted in the following manner: First, the compasses are set to the radius of the curve under consideration. Then, with the P.C. and P.T. as centers, arcs are described so as to intersect at the center of the curve. Finally, with that intersection as a center and the same radius, the required curve is drawn between its P.C. and P.T. For example, the 6° curve whose P.C. is at Sta. $7 + 98.83$ is plotted by setting the compasses to the radius of 955.37 feet; describing arcs with this P.C. and the P.T. at Sta. $13 + 42.72$ as centers; and, with the intersection M of these arcs as a center, drawing a curve from the P.C. to the P.T. The other curves, with their centers at N , O , P , and Q in the order of stationing, are plotted in a similar way.

The curves can be inserted in any order. Obviously, it would be advantageous to draw in all curves having the same radius with one setting of the compasses. Where there is a compound curve, the center of the arc with the shorter radius should lie on the straight line joining the center of the other arc and the P.C.C. Thus, the center P of the arc whose P.C. is at Sta. $33 + 57.79$ should lie on the line from the center Q to the P.C.C. at Sta. $41 + 00.65$.

78. Locating 100-Foot Stations.—It is also required to mark the 100-foot stations along the center lines shown on Plate 303. In Fig. 1, the stations on the tangent from $0 + 00$ to 7

+ 00 are readily located by scaling distances of 100 feet along that line. The P.C. at Sta. 7 + 98.83 is so close to Sta. 8 + 00 that the 100-foot station need not be located. Also, since the lengths of the curves in Fig. 1 are measured in 100-foot chords, Sta. 9 + 00 is located on the curve whose P.C. is at Sta. 7 + 98.83 by scaling a chord length of $900 - 798.83 = 101.17$ feet from that P.C.; and each of the other 100-foot stations on this curve is located by scaling a chord of 100 feet from the preceding 100-foot station. To locate Sta. 14 + 00, a distance of $1,400 - 1,342.72 = 57.28$ feet is measured from the P.T. at

TABLE XV
DATA FOR PLOTTING TANGENTS IN FIG. 2 OF PLATE 803

Tangent		Length	Bearing	Legs of Triangle	
From	To			Parallel to Meridian	Perpendicular to Meridian
0 + 00	5 + 90.61	590.61	Due North		
5 + 90.61	18 + 55.23	1,355.84	N 63° 12' E	5.05	10
18 + 55.23	32 + 07.79	1,391.81	S 72° 27' E	3.16	10
32 + 07.79	39 + 97.57	807.09	S 36° 39' E	10	7.44
39 + 97.57	47 + 52.75	848.98	N 71° 21' E	3.38	10
47 + 52.75	61 + 45.72	1,496.42	N 8° 18' E	10	1.46
61 + 45.72	70 + 50	913.20	N 39° 36' E	10	8.27

Sta. 13 + 42.72. The next three stations are then located at 100-foot intervals along the tangent. Sta. 18 + 00 is plotted by laying off from the P.C. at Sta. 17 + 19.97 a chord whose length is $1,800 - 1,719.97 = 80.03$ feet; and each following station on the curve is established by measuring a 100-foot chord from the preceding station. The other stations on the line are plotted by applying these same principles.

79. **Instructions for Plotting Tangents in Fig. 2**—The tangents in Fig. 2 of Plate 803 are plotted from their bearings and lengths, which are listed in Table XV. These lengths are computed from the tangent distances of the curves and the distances between curves in the manner shown in Art. 75 for Fig. 1. Also, the angle that each tangent in Fig. 2 makes with the meridian is laid off by means of its tangent or cotangent.

On this plate the cotangent is used wherever the bearing angle exceeds 45°. Hence, the longer leg of the right triangle of which the angle is a part is always made equal to 10 main divisions on the 30-scale, and the number of such divisions in the shorter leg is equal to 10 times the tangent or cotangent of the angle. The lengths of the legs of each triangle are shown in Table XV and are indicated in parentheses on the sample plate. Sta. 0 + 00 in Fig. 2 is located $\frac{3}{4}$ inch from the left-hand border line of the plate and 1 inch from the lower border line. The intersection point at Sta. 5 + 90.61 is established on the magnetic meridian through Sta. 0 + 00 at a distance of 590.61 feet, to scale, from the starting point. At Sta. 5 + 90.61 an angle of 63° 12' is laid off by measuring 5.05 units (5.05 main divisions on the 30-scale) northward along the meridian from that vertex to point A and 10 units from A perpendicular to the meridian to point B, and drawing the hypotenuse of the right triangle thus formed. The next vertex at Sta. 18 + 55.23 is then established on this hypotenuse at a distance of 1,355.84 feet from Sta. 5 + 90.61.

To locate the vertex at Sta. 32 + 07.79, a distance of 3.16 units is laid off southward along the meridian at Sta. 18 + 55.23 to point C, a distance of 10 units is measured eastward from C to D, and the hypotenuse of the triangle is drawn from Sta. 18 + 55.23. The required vertex is located on the prolongation of that hypotenuse at a distance of 1,391.81 feet from Sta. 18 + 55.23. The other tangents and vertexes are plotted by proceeding in a similar manner and using the distances shown in Table XV for the legs of the triangles and the lengths of the tangents.

80. **Inserting Curves and 100-Foot Stations in Fig. 2.** After the vertexes of all the curves have been plotted, the next step is to locate all the P.C.'s and P.T.'s by measuring the proper tangent distances from the vertexes, as explained for Fig. 1 in Art. 77. These tangent distances are given in the third column of Table XIII. Then the centers of the curves are located and the curves are inserted as described in Art. 77, the radii being taken from Table XI or XIII.

The 100-foot stations along the tangents and curves are plotted in the manner outlined in Art. 78. Although the distances between stations on the curves are measured along the arcs, it is much more convenient to lay off the chords of those arcs. For the purpose of plotting, it may be assumed that the chord lengths are the same as the arc lengths. The 50-foot stations listed in Table XIII for the 12° curve are located in the field because of the sharpness of the curve, but only 100-foot stations need be shown on the drawing.

81. **Finishing and Lettering Plate 803.**—The final center line in each of the two figures on Plate 803 should be drawn in fairly heavy. It will be found advisable to draw in all the curves first and then to connect the curves by straight lines. If the tangents are drawn first, difficulty is likely to be encountered in getting the curves to touch the tangents properly. The parts of the tangents between the ends of the curves and the vertexes, and also short lengths of the radii at the P.C. and P.T. of each curve and at each P.C.C. should be drawn in; but these lines should not be so heavy as the lines for the curves and their connecting tangents.

The identifying abbreviation, as P.C. or P.T., and the station number of each end of every curve should be lettered along the proper radius, as shown on the sample plate. Also, between the radii through the ends of each curve should be listed the following data for the curve: the value of the intersection angle I ; the degree of the curve D , followed by the letter R or L to indicate whether the curve turns to the right or left; the radius R ; the tangent distance T ; and the length of curve L . The proper arrangement of such data is shown in Fig. 1 on the sample plate. In the case of Fig. 2, it was not possible to insert the data in the right places for the curve between Stas. 14 + 66.02 and 22 + 05.19 and for the curve between Stas. 36 + 50.67 and 42 + 50.67, because it was necessary to indicate the triangles required for plotting the tangents. However, these triangles will not appear on the student's plate, and he should place the data between the radii at the ends of the curves as for the other curves.

The bearing of each tangent should be marked along the part between curves, and the abbreviation P.I. and the station number should be inserted at each vertex. Every fifth 100-foot station on the center line should be numbered by means of a simple whole number, such as 5, 10, 15, or 20. The station numbers of the ends of the lines should also be shown. Either single-stroke inclined letters or single-stroke vertical letters may be used for this work. Then, the meridian arrows should be drawn in approximately the same relative location as on the sample plate. It should be noted that in this case the magnetic meridian, which is represented by the arrow with a half-head, is parallel to the border lines of the plate.

The final work on the plate consists in putting on the title, the figure numbers, the border lines, and the information required in the lower right-hand corner. Upright modern Roman capitals should be used for the title, and single-stroke vertical letters for the figure numbers, the note stating the scale, and the information in the lower right-hand corner. The construction lines shown dotted on the sample plate should be lightly-drawn full lines, but the values in parentheses, and the letters at construction points should not be shown on the student's plate.

over & from

$$\frac{O}{A} = \tan$$

have a hypotenuse over

$$\frac{A}{H} = \cos$$

over Harry's sister

$$\frac{O}{H} = \sin$$

$$\text{opp} = \text{adj} \times \tan$$

$$\text{adj} = \text{Hyp} \times \cos$$

$$\text{Hyp} = \frac{\text{adj}}{\cos}$$

$$(\text{adj}) (\sec) = \frac{\text{opp}}{\sin}$$

Mapping

Serial 2913A-3

PART 1

Edition 1

Examination Questions

Notice to Students.—Study this instruction text thoroughly before you complete the following statements. Read each statement carefully and be sure you understand it.

Now, on your answer paper, list the number of each statement in a vertical column. Immediately after each number, write the letter which refers to the word, phrase, or figure which best completes the statement.

Sample:

(Statement) 1. The sum of 8 and 4 is

A. 2.

B. 12.

(Choices)

C. 14.

D. 32.

(Answer) 1. B

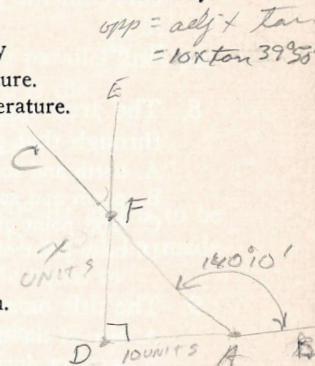
Do not write out the complete statement or the complete answer; just write the number and the letter. Now proceed to answer the first question on this examination.

When you complete your work, examine it closely, correct all the errors you can find, and see that every statement is completed; then mail your work to us. DO NOT HOLD IT until another examination is ready.

1. A graphic scale on a map is affected by
 - A. changes in the size of the map due to moisture.
 - B. changes in the size of the map due to temperature.
 - C. photographic reproduction.
 - D. none of these.

2. True azimuths are measured
 - A. counter-clockwise from a true meridian.
 - B. clockwise from a true meridian.
 - C. counter-clockwise from a magnetic meridian.
 - D. clockwise from a magnetic meridian.

3. In Fig. 12 of the text, if angle BAC is $140^{\circ} 10'$ and is to be laid off by the method of tangents, AD would be made equal to 10 units and DF would be made equal to
 - A. 8.15 units.
 - B. 8.34 units.
 - C. 12.0 units.
 - D. 12.3 units.

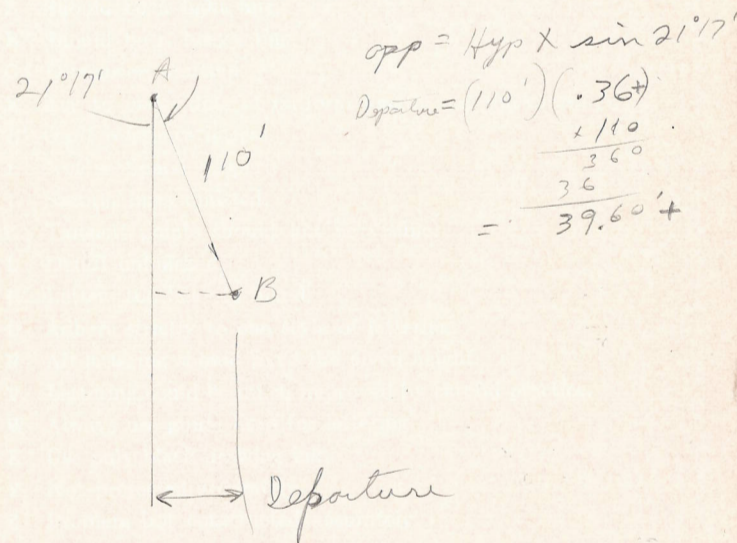


4. If angle BAC in question 3 were $110^\circ 20'$, a convenient way to plot the angle would be to make
- DF equal to 10 units and AD equal to 3.71 units.
 - AD equal to 10 units and DF equal to 3.71 units.
 - DF equal to 10 units and AD equal to 2.70 units.
 - AD equal to 10 units and DF equal to 2.70 units.
5. The length of the courses of a *closed* traverse are usually measured in
- meters and decimal parts of a meter.
 - full and plus stations.
 - feet and decimal parts of a foot.
 - feet and inches.
6. In the modified method of tangents and cotangents, which of the following sides of a square is used as the long leg of a triangle to plot a course that has a bearing of S 40° E?
- North
 - South
 - East
 - West
7. A set of railroad curves is cut to a scale of 100 feet to the inch. If a 9° curve strip fits a curve on a drawing which has a scale of 300 feet to the inch, the specified degree of curve on the ground is
- 3°
 - 6°
 - 9°
 - 12°
8. The true meridian at any point is the line that passes through that point and is directed toward the
- north and south magnetic poles.
 - north and south geographic poles.
 - next point of a closed traverse.
 - next point of an open traverse.
9. The title on a map should include, among other things, the
- size of the field party.
 - weather during the survey.
 - date of the survey.
 - list of field equipment used.
10. The lengths of the courses of an *open* traverse are usually measured in

- chains and links.
 - full and plus stations.
 - feet and decimal parts of a foot.
 - feet and inches.
11. On an ordinary map, the distances shown represent the distances on the ground measured
- with allowance for the earth's curvature.
 - parallel to the ground surface.
 - along contours.
 - horizontally.
12. If one inch on a map represents 200 feet on the ground, the scale of the map is
- $\frac{1}{100}$
 - $\frac{1}{200}$
 - $\frac{1}{1200}$
 - $\frac{1}{2400}$
13. The engineers' scale is so divided that the number of divisions in one inch is a multiple of
- 8
 - 10
 - 12
 - 16
14. The instrument usually used for drawing parallel lines perpendicular to an edge of a drawing board is the
- rolling parallel rule.
 - folding parallel rule.
 - T square.
 - protractor.
15. In Fig. 11 of the text, if angle BAC is $20^\circ 10'$ and is to be laid off by the method of tangents, AD would be made equal to 10 units and DF would be made equal to
- 2.63 units.
 - 2.72 units.
 - 3.67 units.
 - 3.80 units.
16. In Fig. 14(a) of the text, if angle BAC is $20^\circ 30'$ and is to be laid off by the method of chords, AD would be made equal to 5 units and ED would be made equal to
- 1.75 units.
 - 1.78 units.
 - 3.50 units.
 - 3.56 units.

17. In Fig. 18(a) of the text, if the survey starts at station A and angle AOB is $130^\circ 10'$, the *deflection angle* is
 A. $49^\circ 50'$ L C. $130^\circ 10'$ L
 B. $49^\circ 50'$ R D. $130^\circ 10'$ R
18. In plotting a traverse, an error in the direction or length of one course will NOT affect the courses that follow, if the traverse is plotted by the method of
 A. deflection angles. C. chords.
 B. tangents. D. coordinates.
19. The *degree* of a simple circular curve is measured by the
 A. total central angle.
 B. deflection angle at the P.I.
 C. angle at the P.C. from the P.I. to the P.T.
 D. central angle subtended by a 100 ft arc or chord.
20. In Fig. 25 of the text, the tangent distance of the first curve EF is 387.21 ft and that of the second curve GH is 210.96 ft. The distance between the P.I. at B and the P.I. at C is 922.65 ft. What is the distance between the P.T. at F and the P.C. at G?
 A. 148.23 ft. C. 500.73 ft.
 B. 324.48 ft. D. 535.44 ft.
21. A distance of 7.6 chains is equal to
 A. 45.6 ft. C. 456.0 ft.
 B. 121.6 ft. D. 501.6 ft.
22. The radius of a $4^\circ 30'$ curve whose degree is measured by a 100-ft chord is
 A. 1273.24 ft. C. 1637.02 ft.
 B. 1273.57 ft. D. 1637.28 ft.
23. If the distance on a map between Sta. 2 + 50 and Sta. 17 + 50 measures exactly 5 in., the scale of the map is
 A. 1 in. = 3 ft. C. 1 in. = 500 ft.
 B. 1 in. = 300 ft. D. 1 in. = 1500 ft.

24. When laying off an acute angle by the tangent method, the work is simplified if the base of the construction triangle is made equal to
 A. 10 divisions of the engineers' scale.
 B. 10 divisions of the architects' scale.
 C. 12 divisions of the architects' scale.
 D. 12 divisions of the engineers' scale.
25. If a course in a traverse is 110 ft long and makes an angle of $21^\circ 17'$ with the meridian, the *departure* of the course is
 A. 24.3 ft. C. 39.9 ft.
 B. 27.6 ft. D. 102.5 ft.



KEY TO CRITICISMS ON MAPPING

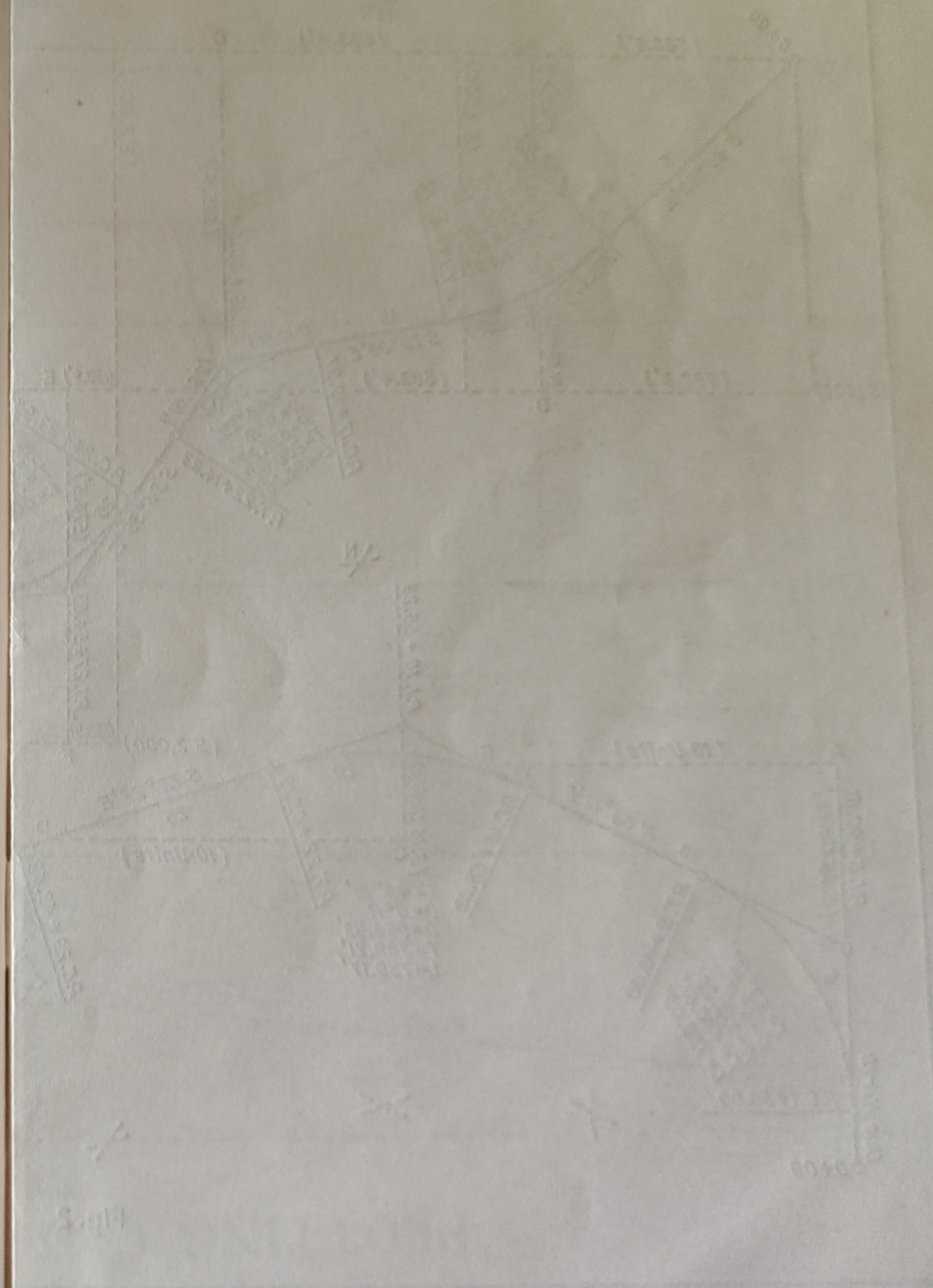
Many students do not desire to have our comments written in full on the Plates. We have, therefore, adopted the following system to indicate briefly our criticisms and suggestions for the improvement of the work. For example, the letter **a** written near a line on the Plate means that the line is rough and uneven

- A** Should be a full line.
- A'** Print all explanatory notes.
- B** Should be a light line.
- B'** Should be a heavy line.
- C** Dimension omitted.
- E** Try to make dots of uniform length and thickness.
- H** Omit reference letters.
- I** Not sectioned properly.
- J** Section lines omitted.
- K** Tangent points should not be visible.
- L** Detail unfinished.
- N** Letters too heavily inked.
- Q** Adhere strictly to one style of lettering.
- R** All lettering should have the given height.
- V** Lettering could be much improved by careful practice.
- W** Always use guide lines for lettering.
- X** Use only black drawing ink.
- Y** Not projected accurately.
- Z** Problem not constructed accurately.

KEY TO CRITICISMS ON MAPPING

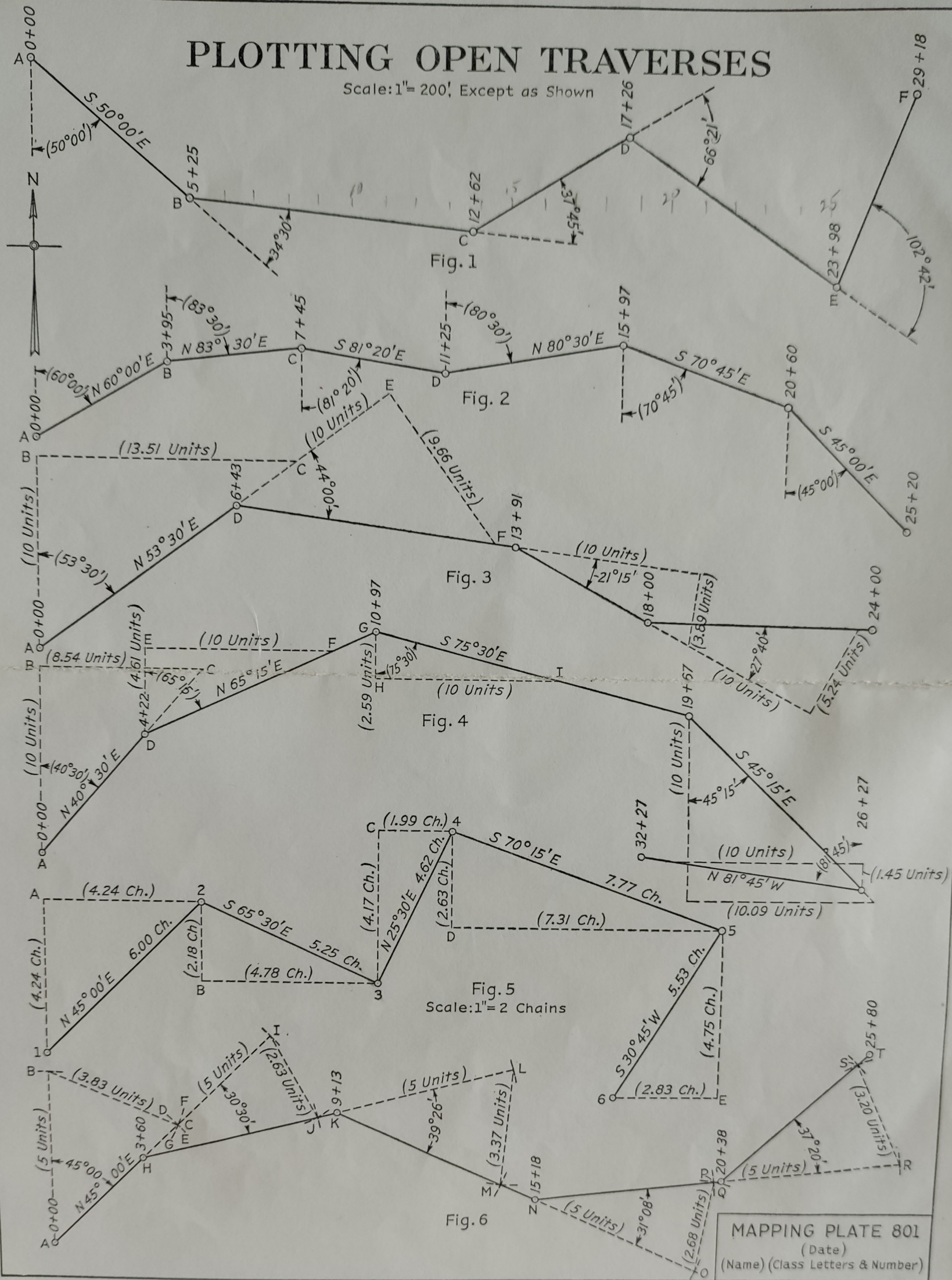
(Continued)

- a Line rough and uneven in thickness.
- b Line apparently drawn freehand.
- b' Avoid double lines.
- c Similar lines should have the same thickness.
- c' Letters and numbers should be printed, not written.
- h Wrong dimension.
- i Given line not made to exact length stated.
- j Lines should pass through exact point of intersection.
- k Line not spaced evenly.
- m Border lines should be heavy.
- n All center lines should be shown.
- o Dots too long. Make them as near $\frac{3}{16}$ " long as possible.
- p Dots too short. Make them as near $\frac{3}{16}$ " long as possible.
- q Dots are evidently inked freehand.
- t Blots should be carefully erased.
- u Section lining not evenly spaced.
- v Arrowheads are too wide and flaring.
- w Arrowheads poorly formed.
- z Not laid off accurately.
- / Letters and figures marked thus / on the Drawing are incorrectly formed. For proper formation refer to Instruction Paper.



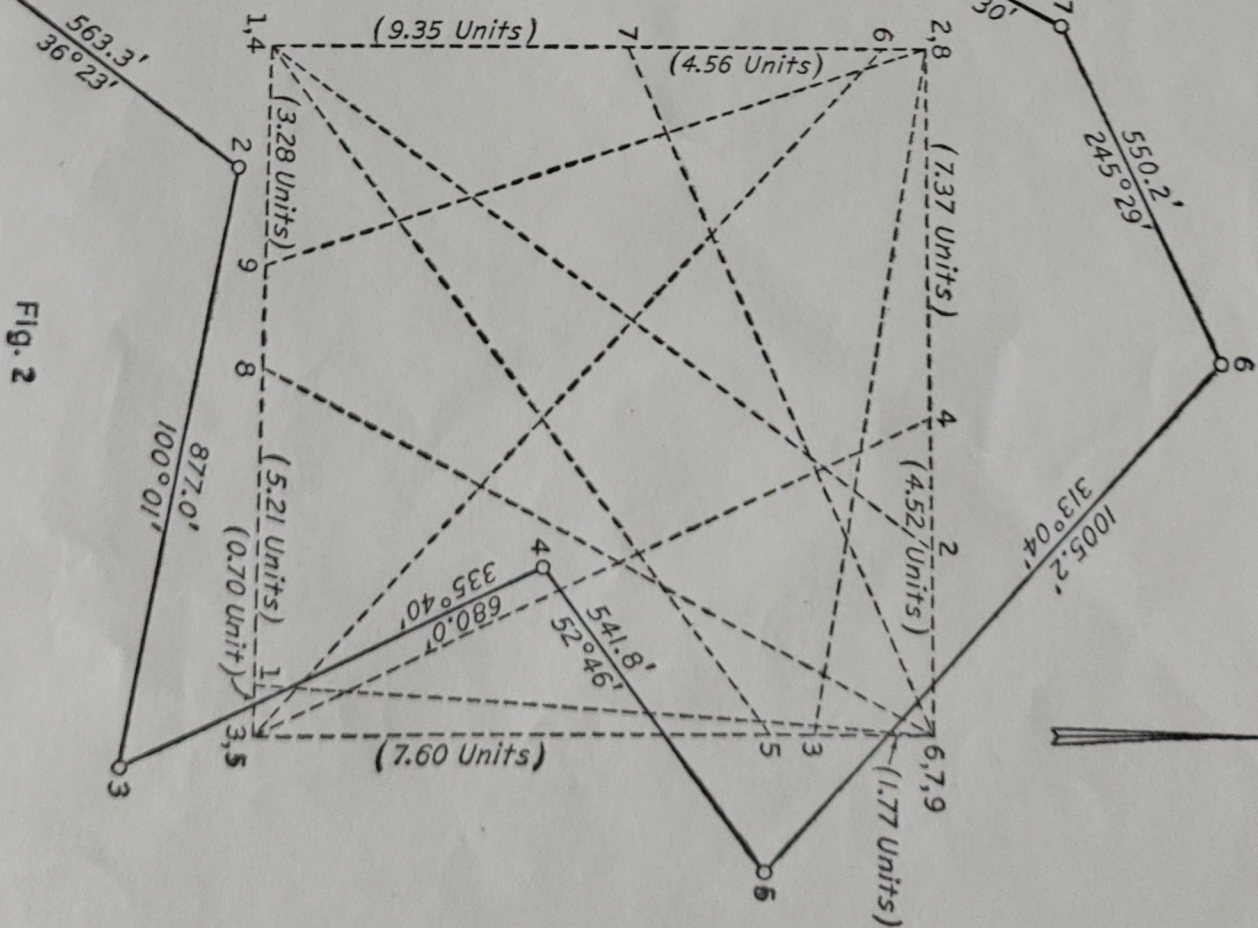
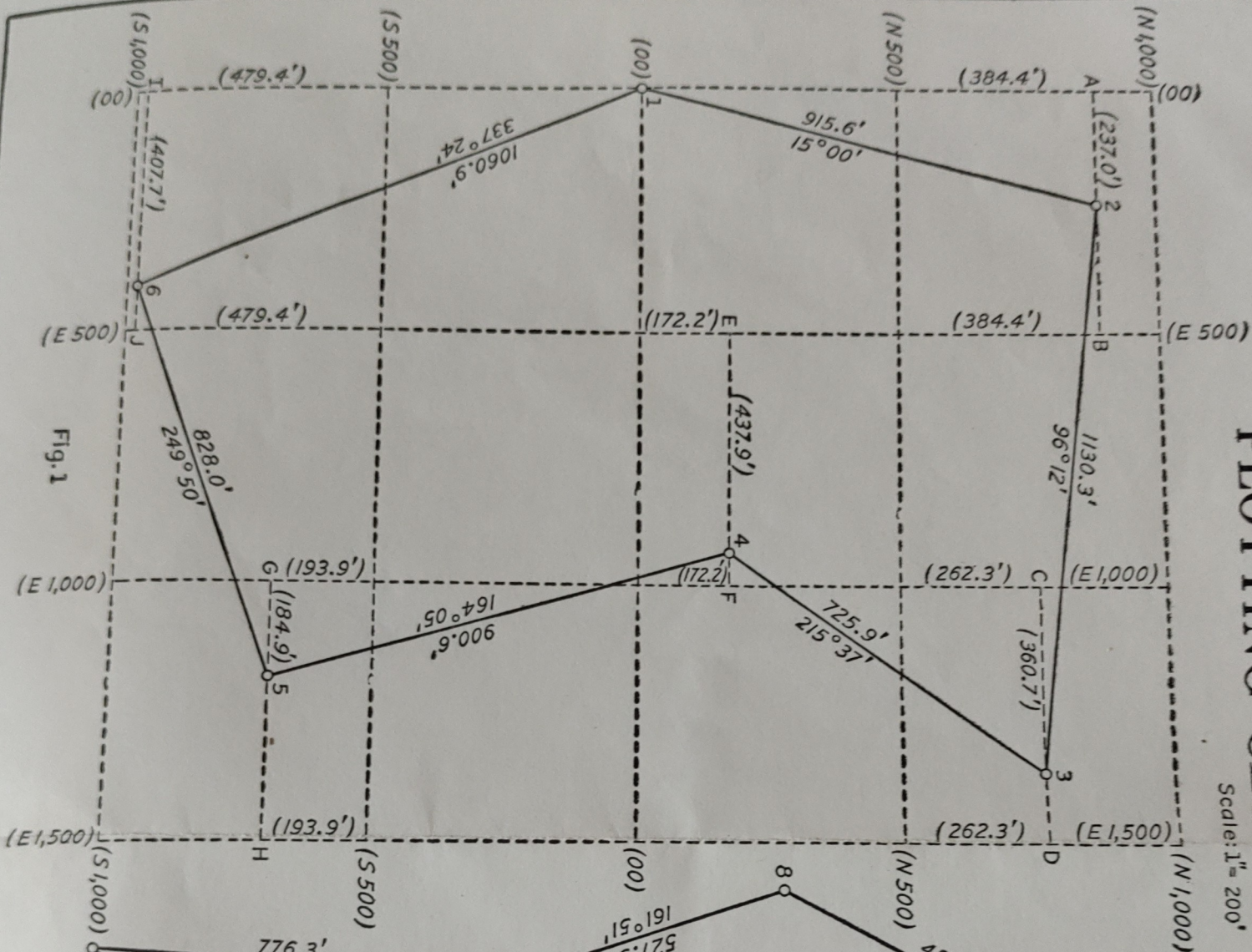
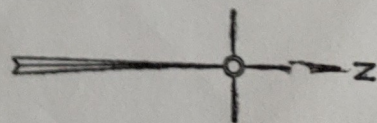
PLOTTING OPEN TRAVERSES

Scale: 1" = 200', Except as Shown



PLOTTING CLOSED TRAVERSES

Scale: 1" = 200'



PLOTTING ROUTE CENTER LINES

Scale: 1" = 300'



Fig. 2

Fig. 1

MAPPING PLATE 803
(Date)
(Name) (Class Letters & Number)